

RESEARCH DEPARTMENT

THE SUBJECTIVE DISCRIMINATION OF
PITCH AND AMPLITUDE FLUCTUATIONS
IN RECORDING SYSTEMS

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1. INTRODUCTION.

The aural effect of the undesired pitch fluctuations which occur in the reproduced output of all types of sound recording systems is well-known. These fluctuations, which are usually described as "wow" and "flutter", arise from imperfections in the recording process, the reproducing process, or in both, and their origin can be traced to imperfect design, manufacture and adjustment of the mechanical (as opposed to the electronic) elements of the system. Report No. C.065, "Wow in Sound Recording and Reproducing Systems", which was issued in July 1948, describes the origin and nature of wow and flutter, and shows that the complex pitch fluctuations occurring in the final output consist, in addition to periodic components due to well-defined imperfections, of a large random element of "Brownian motion" character. The present report is largely concerned with the subjective aspects of wow and flutter and describes work carried out to determine the sensitivity of the listener to these pitch fluctuations. This has involved the development of a pitch fluctuation generator which is novel in that it can produce wow and flutter in "live" programme. The investigation has therefore more reality than previous ones of this type which have been confined to producing fluctuations (by electronic means) in steady tones. In the investigation described in Report No. C.065, and in other investigations, it was found that pitch fluctuations are commonly accompanied by amplitude fluctuations. Tests have, therefore, been included in the present work on the subjective discrimination of amplitude fluctuations, to determine the influence of this factor on the final results. The knowledge gained is necessary for the specification of permissible tolerances for frequency and amplitude fluctuations in recording equipment, a measure which is desirable for promoting general improvement in recording system performance both within the Corporation and in programme exchanges.

2. THE GENERATION OF PITCH FLUCTUATIONS—General.

The complex (undesired) fluctuations of pitch which arise when a recorded programme is reproduced are a combination of the periodic and random fluctuations which have arisen in both the recording and reproducing process¹. The fluctuations result from transient changes in the speed with which the recording medium is moving past either the recording head or the reproducing head. In the recording case this speed change results in a temporary change of recorded wavelength which, on subsequent

reproduction, creates a transient pitch change in the reproduced signal. In the reproducing case the speed change results again in a transient pitch fluctuation as the previously established waveforms move past the reproducing head at a rate different from the recording process. In the disk system the imperfections may arise from eccentricities in the driving system, e.g. in idlers or turntables, from eccentricity of the disk upon the turntable and from lack of flatness in the disk surface. In the magnetic system the imperfections may arise from eccentricities in various parts of the driving system and from changes in tape speed due to frictional variations. In generating intentional pitch fluctuations by mechanical, as opposed to purely electronic means, a fundamental difficulty arises in eliminating, as far as possible, the fluctuations inherent in the mechanical generator itself. For example, a good magnetic tape recorder might be used in which the driving shaft is made intentionally eccentric so that a periodic speed fluctuation is introduced into the recording. The record thus made might be reproduced on another good machine in which the driving shaft is true, with the result that a periodic pitch fluctuation at the frequency of the recording driving shaft will exist in the output. On this periodic fluctuation, however, will be superimposed any random and unintentional periodic fluctuations arising in the recording and reproducing systems. Thus, subjective tests carried out in this manner cannot be said, without positive supporting evidence, to provide an accurate assessment of the subjective effect of the driving shaft fluctuation alone. In addition, of course, the provision of a wide range of fluctuations (in both magnitude and frequency) by this or similar means, would involve prodigious labour. It is fundamental difficulties of this kind which have confined previous investigations to the generation of pitch fluctuations by electronic means. In general a regular pitch fluctuation may be introduced into a tone by the periodic alteration of the value of some oscillator circuit element. No similar method is known by which there can be created the same percentage of frequency modulation of all component frequencies in a complex programme signal. A characteristic of the frequency modulation (or pitch fluctuation) which arises from the imperfections in recording systems is that all component frequencies in a signal undergo the same percentage modulation at the same time. This is to be expected, for a given percentage change of speed in the recording process must alter all recorded wavelengths by the same percentage, and a given speed change in the reproducing process must increase the speed of all recorded wavelengths past the reproducing head by the same percentage. Hitherto then, the mechanical problems have forced investigators to the use of electronic apparatus and this, in turn, has confined the investigations to steady-tone tests. Recently, however, some effort has been devoted in Research Department to the development of a technique of recording (for short-storage purposes) on a magnetic disk rotating out of contact with its recording, reproducing and erase heads. This technique has made possible the development of a controlled flutter generator which has negligible inherent flutter of its own but in which a wide range of controlled flutters can be introduced, as desired, into the reproduced output. This generator will be described in the following paragraphs.

3. THE CONTROLLED FLUTTER GENERATOR FOR PROGRAMME.

3.1. General.

In the Appendix a mathematical analysis of flutter generation is given in general terms which are applicable to the generator to be described in the following

paragraphs, and to similar systems. The mechanical design of the present flutter generator is fairly simple so that the inherent (accidental) flutter due to imperfections of manufacture may be made small. In addition, however, the analysis indicates that design parameters such as the relative disposition of the recording and reproducing heads, and the speed of the recording medium, may be chosen to reduce to negligible proportions, the effect of reasonable manufacturing imperfections. It follows then that the subjective reactions to various controlled (intentional) flutters will not be coloured by the presence of appreciable accidental components. Detailed calculations and measured values of the accidental flutter components present in the output of the generator are given in Section 3.6.

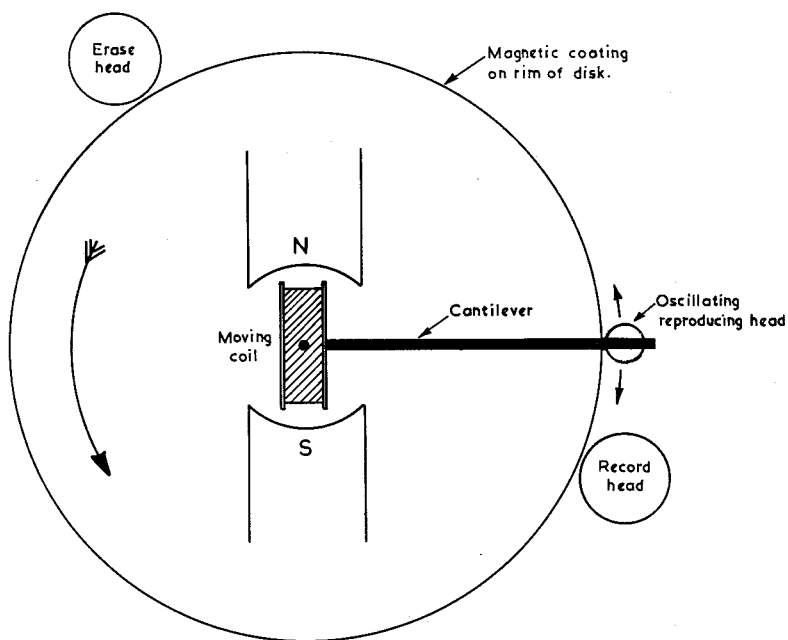


Fig. 1 - Plan of flutter generator for programme

3.2. Description of Equipment.

The mode of operation of the flutter generator for programme is illustrated by Fig. 1. A rigid disk, composed of non-magnetic material, is rotated at a constant speed in a plain bearing. The rim of the disk is coated with a mixture of magnetic oxides similar to that which is normally applied to the plastic backing in magnetic tape manufacture. As the endless magnetic track rotates it passes, in turn, an erase head, a recording head and a reproducing head. Each of these heads is separated from the magnetic coating by a distance of about 0.0005 inches (0.013 mm). Now when the track is separated from the recording head there is a drop in recorded level as the magnetising field strength in the medium decreases. This loss is not too serious, however, for small separations and can be made up by a reasonable increase of bias and recording currents. The loss of output F which occurs when the reproducing head is

separated from the medium by a distance "d" is given by the expression²

$$F \propto \exp(-2\pi d/\lambda)$$

This is much more serious, for it represents a loss of about 55 db per wavelength separation. At short wavelengths therefore the effect of even small displacements may be very great. It follows that when recording and reproducing heads are working out of contact with the disk, the recording speed must be increased well above normal (so that the wavelengths of the higher frequencies are much increased) and every care must be taken to keep the separation as nearly constant as possible to avoid unwanted amplitude modulation. In the manufacture of the disk and its bearing system special precautions have therefore been taken to ensure concentricity so that as the disk rotates the distance between the surface of the heads and the magnetic coating does not change by an appreciable fraction of the shortest wavelength to be recorded. The peripheral speed of the disk is approximately 100 inches (254 cm) per second. The reproducing head, which will be described in detail later, is of a light, miniature construction and is mounted on the end of a duralumin cantilever which is rigidly fixed to the centre of the moving coil in a conventional moving coil system. The coil, and hence the cantilever and head, can move about a central vertical axis which is coaxial with the axis of rotation of the magnetic disk. Suppose now that the disk is rotating and that a programme signal is fed into the recording head, together with the necessary bias current. As long as the reproducing head is stationary the recording track will be moving past it at a constant speed, ignoring for the present any inherent speed fluctuations due to imperfections in the manufacture of the wheel or its driving system. If, however, a low-frequency alternating current is now fed into the moving coil it will oscillate round the central axis, and the end of the cantilever, on which the head is mounted, will then move to and fro. The amplitude and frequency of this movement will depend upon the amplitude and frequency of the

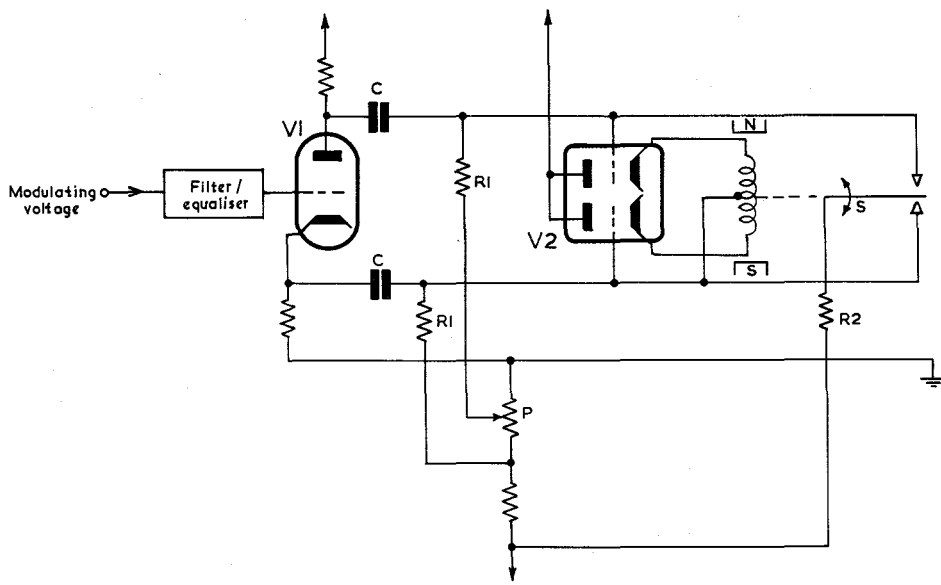


Fig. 2 - Circuit of moving coil drive system

exciting current which is fed into the moving coil. There will thus be a periodic fluctuation in the relative speed of the disk past the reproducing head so that a wow or flutter will be created which depends on the exciting current and the length of the cantilever. Such a system is possible only when the head is working out of contact with the magnetic medium. Since the moving coil is rotating coaxially with the disk, the separation between the reproducing head and magnetic disk remains nominally constant whatever the amplitude of oscillation of the cantilever system. Thus no amplitude modulation accompanies the flutter produced by this generator apart from that which is due to any residual eccentricity of the disk itself.

3.3. The Moving Coil Drive System.

The essentials of the electrical arrangement for the moving coil drive are shown in Fig. 2. Since response down to 0.5 c/s is necessary, direct coupling is employed except for the penultimate stage. The centre-tapped coil L is fed in balance from the cathode followers V_2 . Preceding this stage is a phase splitter V_1 , into which the modulating e.m.f. is fed through correcting networks of suitable response. The balance of currents in each half of the coil can be controlled by the potentiometer P, which adjusts the bias on one output valve and serves to centre the rotating head. If the horizontal cantilever attached to the coil were constrained to a centre position by mechanical compliances, resonances would tend to occur within the working range. Slow drift of the zero position is corrected by a simple step feedback system. A fine spring wiper-arm is connected to the cantilever, and moves across two adjacent contacts separated by a thin sheet of insulation. The wiper is connected through a high resistance to a negative bias source, and the two contacts returned to the grids of the output valve. Should the coil drift from centre, the wiper touches one of the contacts so that R_2 is connected in parallel with R_1 across one of the grids, and the potential at the grid changes with a time constant dependent on the values of C, R, and R_2 , reducing the current in one half of coil L, and slowly restoring the central position. The correcting motion can be made sufficiently slow to be of no consequence to the output, since the wow magnitude is velocity dependent, and furthermore, relatively hard to detect subjectively when the rate is slow.

The disposition of the magnetic heads around the disk is of some importance. It is shown in the Appendix that the recording head and the reproducing head should be close together to reduce the effect of any unwanted accidental flutter in the system. The reproducing head cannot be conventionally screened otherwise mass is added to the cantilever arm and the inertia rises excessively. Special precautions must therefore be taken in screening the recording head and the erase head, and the latter is moved to the opposite side of the disk, as far away from the reproducing head as possible. The permanent magnet of the moving coil system must also be totally enclosed in a steel box to prevent its leakage flux affecting the recording system and causing an increase in noise.

3.4. The Reproducing Head.

To attain minimum inertia of the moving-head system the reproducing head must be made as light as possible whilst maintaining adequate sensitivity for out-of-contact working. The conventional core, consisting of a stack of mumetal laminations, would be too heavy for use in the system and a single lamination head which is

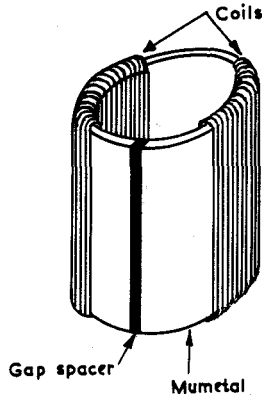


Fig. 3 - Single lamination reproducing head

illustrated in Fig. 3 is, therefore, employed. In this head a single rectangular lamination is sprung into a circular form and held in position by a light clamp made of plastic material. The gap of the head is formed by the line of contact between the two ends of the lamination which are held at a suitable distance apart by a spacer. Two coils are wound around the lamination, as shown in the diagram, suitable spaces being cut in the plastic clamp to accommodate them. The coils of the head are connected to the external circuit by fine wires, which do not appreciably affect the mechanical stiffness of the system. The frequency response of this head can be made almost identical with that of good conventional heads, but its sensitivity is less.

3.5. The Magnetic Disk Driving System.

The magnetic disk has a large inertia of its own and this is increased by a fly-wheel mounted underneath it. The magnetic disk and fly-wheel are driven, through a highly compliant belt, by a synchronous motor rotating at 1,500 r.p.m. The compliance of the belt and the combined inertia of the drum and fly-wheel together produce a low-pass filter system which confines the range of frequencies transmitted to the drum from the driving system mainly to the 1 c/s region. A consideration of the accidental flutter occurring in the apparatus is given in the next paragraph.

3.6. Accidental Frequency Fluctuations in the Generator.

It is shown in the Appendix that the ratio of the instantaneous frequency, f_r , emerging from the flutter generator to the frequency, f , fed into it is given by the equation

$$f_r/f = 1 + (\omega vs/v_0^2) \cos \omega(t + T_0/2) - (v_1/v_0) \cos \omega_1(t + T_0)$$

where s is the mean distance between the recording head and the moving reproducing head,
 v_0 is the constant component of the recording medium velocity,
 $v \sin \omega t$ is a representative accidental component of recording medium velocity (causing accidental flutter),
 $v_1 \cos \omega_1 t$ the velocity of the moving reproducing head (causing intentional flutter),
 and the time T_0 is given by s/v_0 .

If the generator is perfectly made, then $v = 0$ and putting this value in the equation gives simply

$$f_r/f = 1 - (v_1/v_0) [\cos \omega_1(t + T_0)]$$

which is the desired result for a flutter generator. In practice, however, v is finite for there are always some small irregularities of traction even in well designed systems. The irregularities are mostly of a random nature and with superimposed periodic variations due to motors, idlers, etc.¹. The first equation above suggests how these accidental fluctuations can be minimised in the system described. The second right-hand term of the equation represents the uncontrolled variations, and it will be seen that the effect of a given value of v depends on the factor $\omega s/v_0^2$ and so for minimum accidental fluctuations in the output it is necessary to make

- (a) the velocity, v_0 , of the recording medium, large
- (b) the separation, s , of the heads, small
- (c) the angular frequency, ω , of the accidental speed variation, low.

In the flutter generator described the peripheral velocity of the drum is some 100 inches/second with a drum radius such as to give a fundamental value of ω of about 20 radians/second. The minimum value of $s = 1.75$ inches was set by head dimensions. The factor $\omega s/v_0$ gives the reduction of fundamental wow from any drum eccentricity as being 0.35. A greater reduction factor is unnecessary, for the eccentricity which can be tolerated is even more severely limited by the amount of amplitude modulation which arises due to varying separation of the reproducing head. More important are the speed variations at 25 c/s and other frequencies from the driving motor, etc. These, as previously described, are eliminated by driving the drum, which has considerable inertia, by means of a highly compliant belt. This low-pass filter action leaves only fluctuations with frequencies predominantly in the region of 1 c/s and these are effectively reduced by the correlation mechanism considered. With the same constants as before, except for $\omega = 2\pi$, the reduction factor is 0.11.

Actual measurements of undesired fluctuations from the drum under working conditions gave a value of 0.03% peak. These fluctuations consisted of a roughly equal mixture of random components and a regular wow at the fundamental frequency of rotation. This small value is of no consequence from the subjective viewpoint.

4. ELECTRONIC GENERATORS.

4.1. The Electronic Flutter Generator (Tone Tests).

A portable tone source was used to produce wow and flutter on pure tones, with the modulating signal injected in place of the fixed warble tone normally produced in the instrument. In essence the modulating signal was made to vary the gain of an amplifier controlling the capacitance presented to a tuned circuit, by means of the "Miller effect", and hence to vary the frequency of the beat frequency oscillator. Sinusoidal modulating signals down to 0.5 c/s were externally generated by a "Wien bridge" type of oscillator, whilst square and pulse waveforms were produced by a multivibrator.

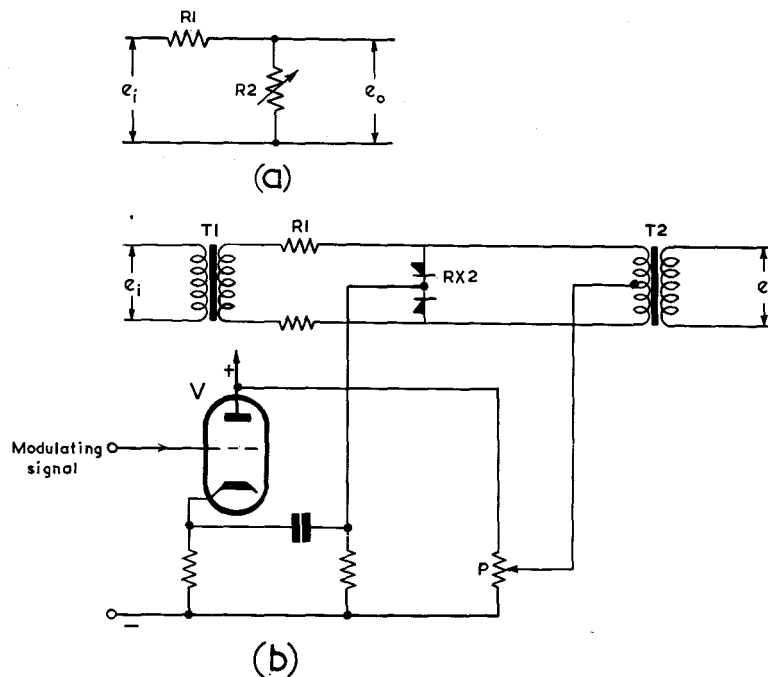


Fig. 4 - Circuit of amplitude modulation generator

4.2. The Amplitude Modulation Generator (Tone and Programme Tests).

To simulate amplitude fluctuations on recorded programmes, it is necessary to amplitude modulate a "carrier" (the programme) which has an extensive spectrum, and to do this in a manner which produces the necessary sidebands without significantly distorting the "carrier" or adding the modulating function to the output. Fig. 4(a) shows a basic method of achieving this result. The programme e_i is fed to a potentiometer comprising R_1 and R_2 , the value of R_2 being a function of the modulating signal. R_2 could be varied by mechanical means, and the output signal e_o would then be modulated as desired.

A modification of this, the basic circuit actually employed, is shown in Fig. 4(b). The mechanical variable R_2 in Fig. 4(a) is replaced by a rectifier RX_2 , whose resistance is a function of the e.m.f. applied to it. In addition to the resistive change, the modulating e.m.f. itself appears across the output of RX_2 . This can be eliminated by using the balanced arrangement shown and the output from T_2 then contains only the wanted products. The modulating signal is fed from cathode follower V , to the junction of the rectifiers and mid-point of the primary winding of T_2 , the potentiometer P adjusting the standing potential on the rectifiers to produce the best working conditions. When the relative values of alternating and direct potentials are suitably proportioned, sensibly linear modulation up to about 25% can be obtained, but great care must be taken to avoid both resistive and reactive unbalances if the modulating signal itself is to be kept below audibility threshold. The calibration of the modulator presents no special difficulties.

5. CALIBRATION OF PITCH FLUCTUATION GENERATORS.

5.1. The Magnetic Disk Flutter Generator.

The degree of amplitude modulation of a signal can be measured without much difficulty; the assessment of frequency deviation, however, requires care, especially when, as in recording systems, a component tone may be modulated at a rate in excess both of the deviation and frequency of the tone itself. This is a condition far removed from normal frequency-modulation practice. Fortunately wow and flutter are by their stochastic nature, phenomena such that in practical cases the frequency deviations are small. Thus the notion of "instantaneous frequency" can be usefully employed and approximate solutions to the nature of the resultant waveforms obtained.

The calibration of the magnetic disk flutter generator is easily accomplished since speed variations can be deduced from measurements centred on one particular frequency. For a given speed change the absolute frequency change in a recorded tone is proportional to the frequency of tone and a fairly high frequency is, therefore, convenient for calibration. If too high a frequency is adopted, however, the amplitude modulation resulting from disk eccentricity and out-of-contact working, together with general system noise, present difficulties. A nominal calibration

frequency of 3 kc/s was therefore chosen and any amplitude modulation present was removed by a simple diode limiter. The calibration was carried out by applying the 3 kc/s tone, in which flutter has been introduced by the generator, to a discriminator network which transformed the frequency variations into amplitude variations. The disk flutter generator is not used to produce a flutter frequency in excess of 100 c/s so the discriminating network is not called upon to perform a linear amplitude conversion above this frequency*. In this case a conventional

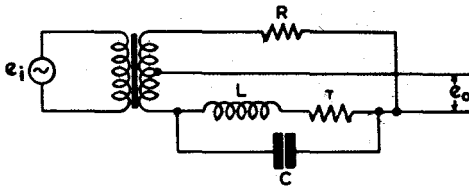


Fig. 5 - Discrimination for lower range of flutter frequencies

"phase" discriminator of the type shown in Fig. 5 can be used to advantage. Fig. 6 shows that the performance of this discriminator is adequate if the half-bandwidth is 150 c/s, and mistuning 50 c/s. In this condition the amplitude modulated response at a flutter rate of 100 c/s and 1.5% magnitude is in error by 1 db and frequency deviations of 0.1% produce amplitude modulation of the order of 10%. Measurement of the rectified output of the discriminator enabled the speed variations of the cantilever-head system to be related to the e.m.f. applied to the moving-coil drive system. The flutter region calibrated lay between $\frac{1}{2}$ c/s and 100 c/s at magnitudes up to 1.5% and the ratio of velocity change to driving e.m.f. was found to be sensibly linear except for changes of less than 0.2% at frequencies below 1 c/s. In the latter region friction in the pivot system became an appreciable factor, but in practice such conditions were not used, being below subjective threshold.

*The behaviour of discriminators is, in general, extremely complicated, and the question will be examined in more detail in another report. It will there be shown that the familiar dilemma of "speed versus resolution" is in evidence and that attempts to increase the sensitivity of amplitude conversion for small deviations entail an unwanted discrimination against higher flutter rates.

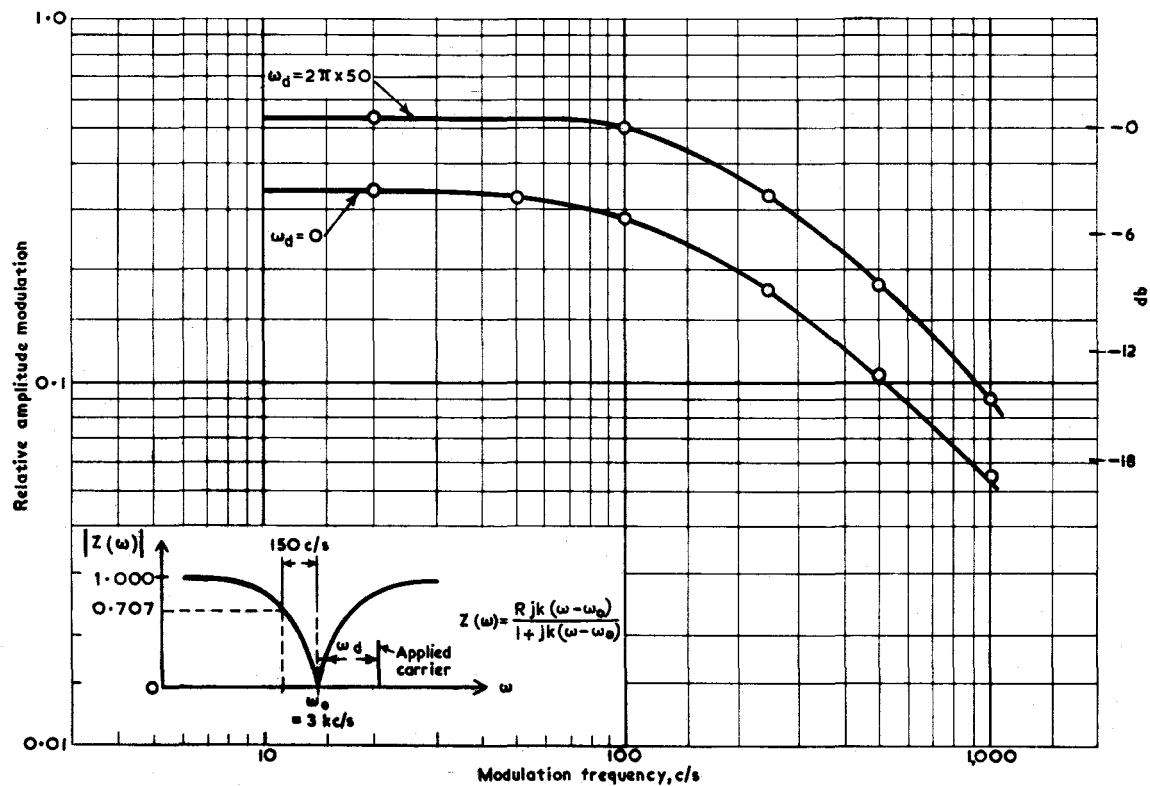


Fig. 6 - Output of discriminator at various modulating frequencies

The calibration procedure for impulsive or square-wave modulation of the recorded signal was exactly the same, for the flutter waveforms were spectrally restricted to a few hundred cycles by the electro-mechanical behaviour of the system and the discriminator frequency response was therefore still quite adequate. In certain experiments the moving reproducing head system was driven by a source of random noise, and a pen recording was taken of the discriminator output. The frequency deviations were, in this case, of a statistical nature, and the measurements were carried out, and results analysed, in the manner suggested by Axon and Davies¹. The distribution of frequency deviations measured from the ordinates of the pen-recording were closely described by a form of Poisson relationship,

$$q(R) = N/2X^3 \cdot R^2 e^{-R/X}$$

where $q(R)$ is the number of occurrences of a frequency fluctuation of magnitude R and N is the total number of fluctuations recorded. The mean of the distribution has a value of $3X$ and the mode a value of $2X$. Thus the probable frequency excursions can be related to the level of noise fed to the drive system.

5.2. The Electronic Flutter Generator.

Tests with pure tones involve a more extreme range of measurement, for test tone frequencies may range from 50 c/s to 10 kc/s, and modulation rates from 0.5 c/s to 10 kc/s. Thus the discriminator used for calibrating the magnetic disk generator was of little use.

However, it can be shown that use may be made of a differentiating network having an output simply proportional to frequency, with linear phase shift, so enabling a wide range of values to be measured without error. All practical circuits have a cross-over from this ideal regime at some frequency, but this can be made to take place outside the range of significant sidebands.

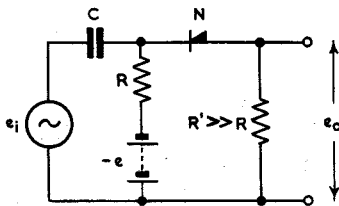


Fig. 7 - Discriminator circuit for higher range of flutter frequencies

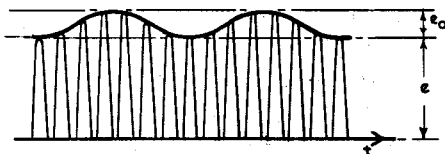


Fig. 8 - Voltage relations in discriminator circuit

The sensitivity of such an arrangement is unfortunately small, giving only a frequency-to-amplitude-modulation conversion of unity. Since the effects to be measured are of the order of 1%, the reading of such small signals must be facilitated without recourse to frequency-dependent networks. This is accomplished by a circuit which is shown in essentials in Fig. 7. The signal e_i after passing through the differentiating network CR is fed to the diode N which cuts off signals below a value of e . This value is adjusted (see Fig. 8) to be just below the minimum modulated input. The emerging voltage e_o is thus modulated by a large percentage, and is more easily amplified and measured. In view of the widely varying ratios of carrier to modulation frequencies that are possible, the measurement is conveniently carried out by means of a scale on a cathode ray tube, display-

ing e_0 on a suitable time axis. Without this visual display there are measurement difficulties when the modulation frequency approaches, or becomes greater than, the carrier frequency.

6. DISCUSSION OF EXPERIMENTAL PROCEDURE.

6.1. General.

The aim of the experiments is to determine when modulation of the test signal is first noticeable, i.e. the subjective thresholds are measured under chosen conditions and are not necessarily associated with annoyance value. In practice the effects of modulation may not always be displeasing from an aesthetic viewpoint. The experiments fall into four broad classes, namely the amplitude or frequency modulation of test material consisting of tones or broadcast-type programme. Although a prime object is to establish tolerable degrees of wow and flutter under practical conditions, such a wide range of possibilities exists that it is impossible to arrive at any single figure of merit which covers all possible conditions. The effect of wow or flutter in particular circumstances may have to be inferred from, or supplemented by a knowledge of, effects which may at first sight appear remote to the question.

Frequency fluctuations are almost always associated with amplitude effects. For example, in any recording system where reproduced output is proportional to velocity, the flutter must be accompanied by an amplitude change. The magnitude of this effect may be negligible, but a much larger degree of amplitude modulation can arise in other ways. It is produced, for example, by listening enclosures and even by the ear itself, whenever frequency modulation is present. Hence it is of interest to determine the subjective threshold of amplitude modulation alone.

Experiments were made, in the course of the work, using both headphones and loudspeakers. The choice of loudspeaker can affect the results, especially in the case of frequency modulation (flutter) of a tone carrier, where local irregularities in the loudspeaker-response curve produce undesired amplitude modulation. The production of harmonics by non-linear behaviour also tends to make the value of threshold lower. Effects of this kind are fortunately largely masked when using a programme source. The loudspeaker unit used for the experimental work described was the LSU/10, and experiments were conducted in the listening rooms at Nightingale Square and Kingswood Warren to simulate typical listening conditions. The reverberation/frequency characteristics of the two rooms are fairly level at lower frequencies in both cases, but they fall gradually at high frequencies. The Nightingale Square room, volume 2,850 cu.ft., has an average reverberation time of about 0.55 seconds, and the Kingswood room, volume 1,890 cu.ft., a reverberation time of about 0.45 seconds. Experiments on a single subject suggest that these differences of reverberation time in the enclosure have no appreciable effect on the results. The wow threshold on piano music was compared for two cases, first as heard under normal conditions, and secondly as heard in an echo room on a different type of loudspeaker. With the proviso that the criterion of "just-noticeable" wow had to be somewhat adjusted in view of the initial jumble of sound (the virgin state could even have been taken as wow by an untrained observer), there was no significant difference of results.

6.2. Tone Tests.

In an early group of experiments to determine the threshold of modulation effects on pure tones, the signal was presented to the subject through high-grade moving coil headphones. The headphones were subjectively calibrated for loudness level in the audio range, and the level chosen (unless otherwise stated) was 75 phons, which corresponds to the preferred peak listening level for light music programme.³ Test tones of various frequencies between 50 c/s and 10 kc/s, modulated at frequencies unknown to the subject, were presented in random order. Various discrete levels of modulation of a test tone were presented, and the subject asked if the modulation was audible or not. The results with any subject were found to be surprisingly inconsistent and it was soon realised that aural fatigue and auditory imagery were serious factors in these circumstances. The ear rapidly tired of the repetition of a pure tone, and furthermore, once a given rate of modulation had been heard, it gave rise to an after-image which was often most pronounced. Even if a pure tone was presented afterwards, considerable "hysteresis" was evident, the modulation still being "heard in imagination".

It was found that greater consistency resulted if the *pure* tone was first presented and the modulation gradually increased until the subject indicated he was aware of a change. On the account of the hysteresis effect, no attempt was made to repeat the test on a descending scale and the tone was cut quickly upon recognition of the modulation, in order to avoid fatigue. A new test frequency then followed, together with a new modulation frequency. This technique seemed to reduce considerably, the fatigue of the subject and the aural imagery (possibly due to psychological rather than physical factors) seemed to be reduced by the subject's knowledge that the next rate would be different, even though he did not know precisely how different.

The same method was also employed in other experiments, using a loudspeaker in place of the headphones, when it was not desired to eliminate the effect of room eigenmodes.

6.3. Programme Tests.

In the experiments with musical programme in listening rooms, a small group of subjects was tested at one time. Since it was impossible in these circumstances to arrange for the peak programme level to be exactly 75 phons for each subject, the seating was arranged to give roughly an equal scatter above and below this figure. In such rooms, however, the general loudness does not change rapidly for slight changes of position and a greater change is probably that due to non-axial frequency response of the loudspeaker.

Subjects who did not understand the term "wow" and "flutter", and who were not well acquainted with their effects, were allowed to hear typical samples before the tests commenced. Each subject had a switch which could be "made", without the knowledge of the other subjects present, when the occurrence of some disturbance, thought to be due to frequency fluctuation, was heard. This tended to reduce the competitive element and eliminate a "chain-reaction" after the most sensitive listener had pressed his switch. Individual identity was lost by this system, but identical groups could, of course, be reassembled for repeat experiments and individuals could be tested singly when desired.

The presentation of frequency fluctuations in a musical programme is complicated by the fact that there is no steady state of the "carrier" as in the case of pure tone. The detection of modulation of a tone depends on the ear's ability to detect the addition of sidebands to a single line of the audio spectrum, that is to detect an increase of entropy from a state of complete order. The controlling time element of such a performance is psychological. The other extreme of modulating "white noise" gives an entropy reduction from chaos, but a chaos statistically invariant in time as interpreted by aural and psychological mechanisms. A musical programme lies between these extremes, the order-disorder being aesthetically controlled on a time scale of aural significance. It seems reasonable to suppose that for this sort of disorder phenomena, a given degree of fluctuation will be most noticeable at a time when the residual material is most ordered, or uniformly disordered. There are complicating factors, but the general validity of the thesis is borne out by experience. It is well known that a fixed degree of wow is more noticeable on piano music than on speech, indeed a very large fluctuation is necessary sensibly to affect the latter. To fix a tolerable level of wow and flutter for practical purposes it is best to select a type of programme material where the effect is most obvious, so that the effect of the chosen tolerance on other programme material is likely to be negligible. The closest instantaneous approach to an ordered pure-tone regime is achieved by a piano or church organ. In many other instruments, as in the voice, it is common for the performer to introduce vibrato effects which may be mistaken for wow. Thus in the main body of experiments on programme wow a piano solo was used, provided conveniently by an automatic player-piano. The instrument had to be in a very good state of tuning, otherwise beats greater than those inherent in the equal temperament scale could mask the wow effects to some extent.

If the reasoning set out above is correct, it would be expected that a work with fairly long sustained notes or chords would show the effects more markedly than, say, a florid toccata. This seemed to be the case in practice. One of the compositions used was Ireland's "Island Spell", another was a Liszt transcription of Schubert's "Am Meer", edited with removal of the more florid sections. The works presented steady, slow chords, making possible the adoption of the following scheme. A particular waveform and frequency of wow having been selected, the piano programme was modulated in increasing discrete steps of ten to twenty seconds duration. At any particular level there was sufficient time for the programme to pass through a state of easy recognition, and variations of sound level and tone durations were reasonably constant between any group of levels. Any accidental weighting due to the time at which a certain level was presented could be corrected by giving further tests in which the process was repeated with the same time sequence, but in different order. The increase of wow on the programme continued stepwise until the most insensitive listener indicated recognition. Suitable increments in wow level were decided by means of pilot experiments.

As the initial experiments progressed, it became apparent that the subjective threshold of the frequency fluctuations was sufficiently large to permit pre-recording of certain test sequences. A check showed that there was no significant difference in the results given by the same group of subjects when the pre-recording was done on a machine whose fluctuations were of the order of 0.1% peak with waveforms of an irregular nature. During the tests, the number of subjects having "made" their switches at a given time was registered on a meter, and the corresponding fluctuation

level determined on a pre-arranged time schedule of the test recordings. Experiments with amplitude modulation were carried out in essentially the same manner.

7. RESULTS OF AMPLITUDE MODULATION TESTS.

7.1. Pure Tone: Sinusoidal Modulation.

The subjects, eight in number, were all engineers fairly accustomed to listening to such effects and the signals were heard on headphones at a constant level of 75 phons. The mean values of the results for threshold of detection are plotted in Fig. 9. The general appearance of the variation is not unlike the Fletcher-Munson loudness contour⁴ for moderate levels. The ear seems to exhibit regions of enhanced sensitivity around 700 c/s and 4 kc/s with enhanced perception of modulation frequencies around 3 c/s or 4 c/s. At modulation rates below 0.5 c/s comparisons become difficult. Here memory seems to be called into play. The finding that perception is most difficult at rates around 50 c/s was first suspect; it may have been that the engineers had grown accustomed to "mains hum", or that very small amounts of 50 c/s hum were having a masking effect. Such, however, does not seem to be the case, and the phenomenon has now been verified by other workers⁵.

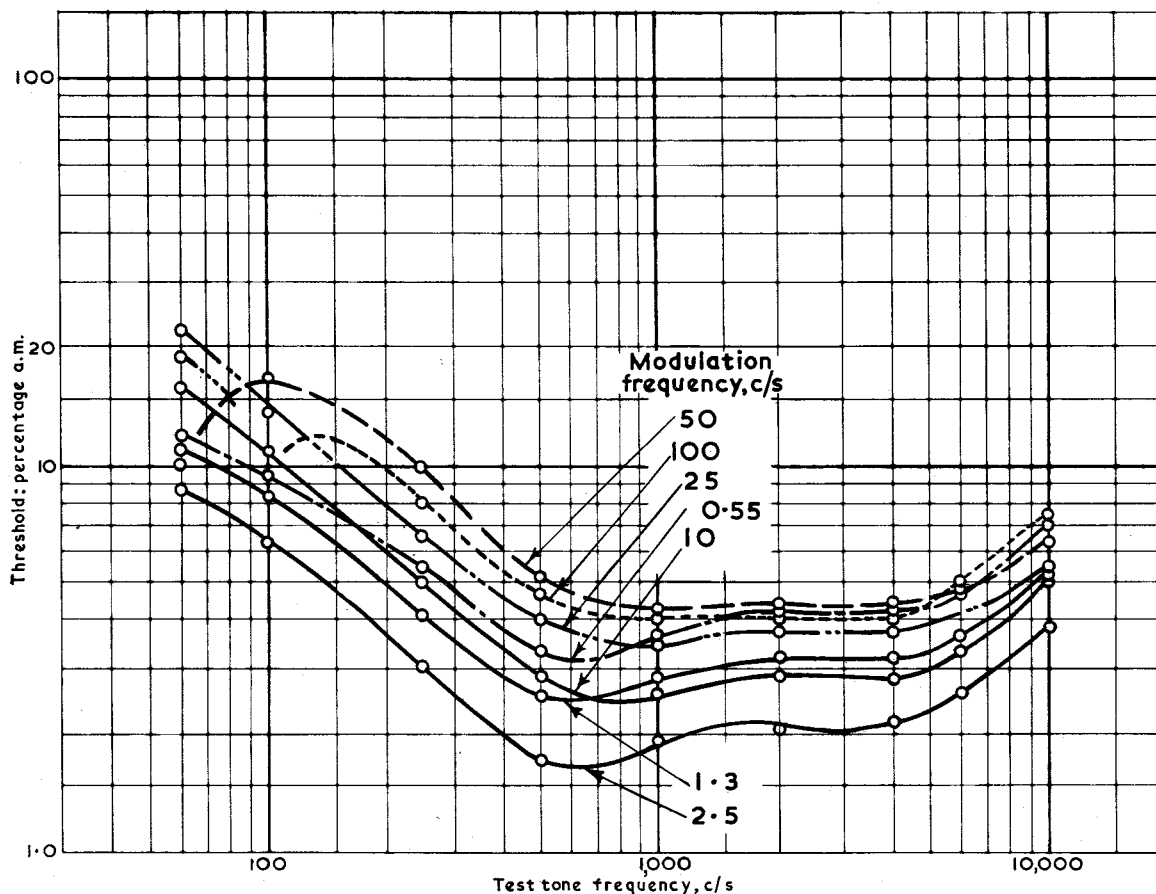


Fig. 9 - Variation of subjective threshold for sinusoidal amplitude modulation of pure tone heard through headphones

The rise of threshold level towards the lower test frequencies which takes place at all modulation frequencies is doubtless related to the decrease of "just-noticeable" loudness steps in the intensity gamut at low frequencies⁶. It will be observed that there is a sudden dip in the curve when, say, 100 c/s tone is modulated at the same rate. This is because the slower beating of frequencies in this area is more easily detected than the higher rate components causing them.

Over a large part of the audio spectrum it will be seen that some 2 to 4% amplitude modulation is necessary for aural detection, depending on the modulation rate. The standard error of the means was of the order of 5% over this region but it rose to about 10% in extreme regions, i.e. when using modulation frequencies of 100 c/s, or 10 kc/s tone modulated at almost any modulation frequency. The means may be even greater in the latter region with a larger population, for it is well known that the perception at high frequencies decreases with age and other factors, and the present experiments used subjects mostly between the ages of thirty and forty years. From these results it is evident that amplitude modulation produced directly in velocity responsive systems is of little consequence, for it will be shown that about 3% of frequency modulation is far more objectionable.

A few observations were made on a tone of 1 kc/s when the frequency of amplitude modulation was extended beyond 100 c/s to the higher end of the audio-frequency region. The mean results for a single subject, which are shown in Fig. 10,

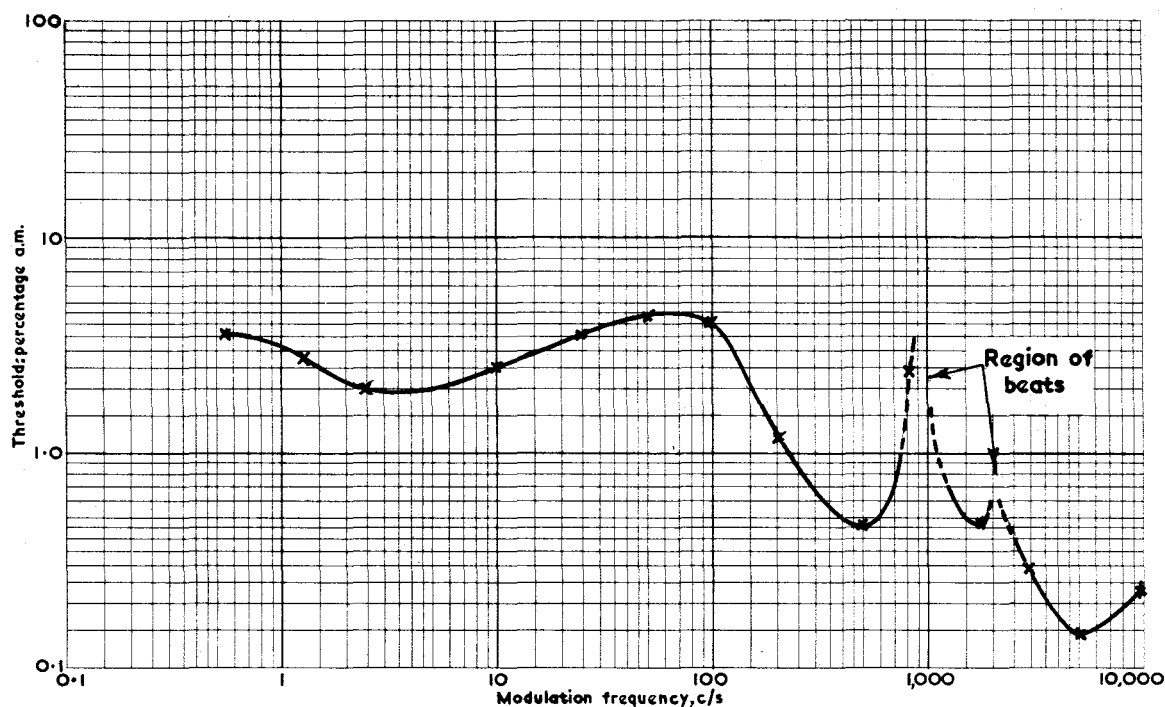


Fig. 10 - Variation of subjective threshold for sinusoidal amplitude modulation of 1 kc/s tone heard through headphones

indicate that the ear is extraordinarily sensitive to changes occurring at rates greater than those normally considered in wow phenomena. Above 100 c/s the threshold falls rapidly so that at a modulating frequency of about 5 kc/s, almost 0.1% amplitude modulation can be detected. There are regions of slow beating around the auto-modulation* frequency and its harmonics, and since the slow beats are less audible than the tone in the region, the threshold is raised locally, a converse effect to that noticed in the first experiments.

7.2. Pure Tone: Square-Wave Modulation.

Only qualitative experiments were carried out on amplitude modulation by waveforms of square or impulsive nature. In this case most of the information is dependent on the precise phasing of the modulating wave with respect to the carrier. When the periods of modulation and test tone are harmonically related a repetitive "crack" is heard, whose loudness is a function of the phase difference, and when the periods are not so related the "crack" waxes and wanes with the changing phase of modulation. It is unlikely that a detailed knowledge of this is of much value in the present enquiry.

7.3. White Noise: Sinusoidal Modulation.

The subjective effect of amplitude modulated noise is of considerable practical interest. In conditions where the recorded signal-to-noise ratio is poor, such as could occur in the "dubbing" of archives, it may well be a most significant factor. The mean results of six tests on "white" noise heard through the loudspeaker LSU/10 at standard level are shown in Fig. 11. It will be noted that the most sensi-

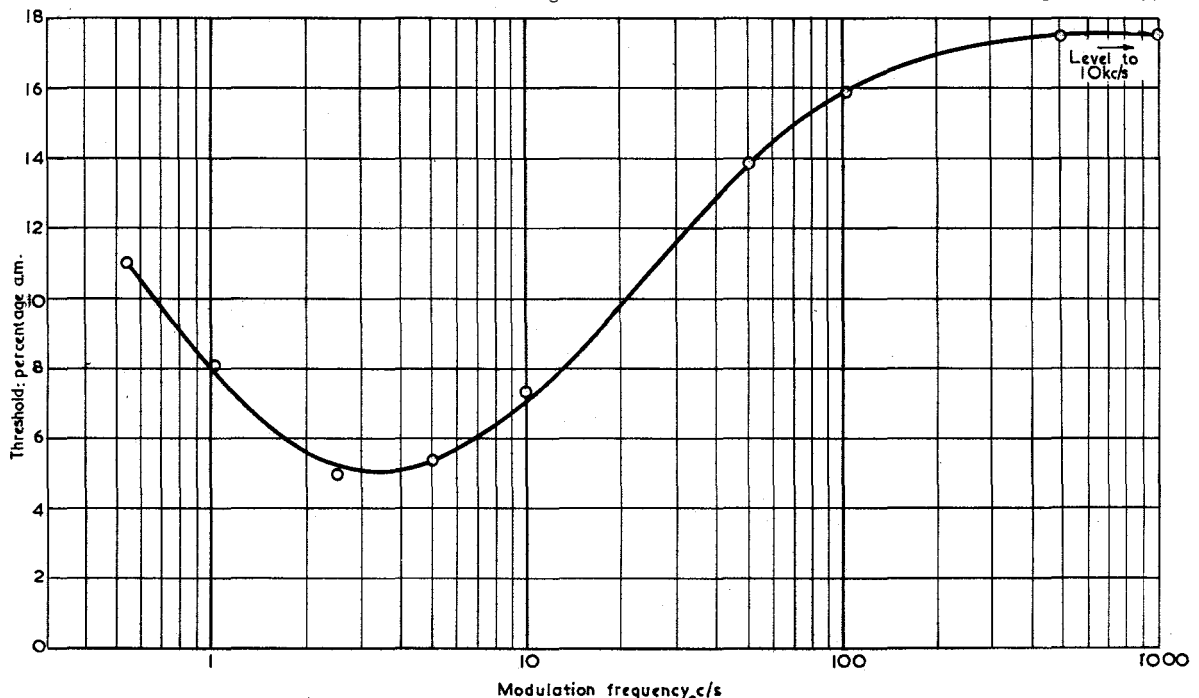


Fig. 11 - Variation of subjective threshold for sinusoidal amplitude modulation of white noise heard through loudspeaker LSU/10

*i.e. When carrier and modulation frequencies are equal.

tive discrimination is in the region of 3 or 4 c/s, as for pure tone, but unlike pure tone the percentage modulation at threshold does not seem to decrease at all above 100 c/s. Some 5% amplitude modulation is required for detection at the most sensitive point which corresponds to the average subjective behaviour with a modulated pure tone of approximately 150 c/s.

7.4. Piano Programme: Sinusoidal and Square Wave Modulation.

To determine the order of magnitude of amplitude modulation which is significant under practical conditions, the threshold was assessed by a dozen subjects listening to piano programme at the preferred level through the loudspeaker. The subjects were again engineers who were familiar, to some extent, with the nature of the effect. The mean results for sinusoidal modulation, which are shown by Curve A of Fig. 12, suggest that between 2 c/s and 20 c/s the rate is not a critical factor, whereas at 0.5 c/s the threshold has about doubled. At still lower frequencies the results must depend on the sense of absolute intensity.

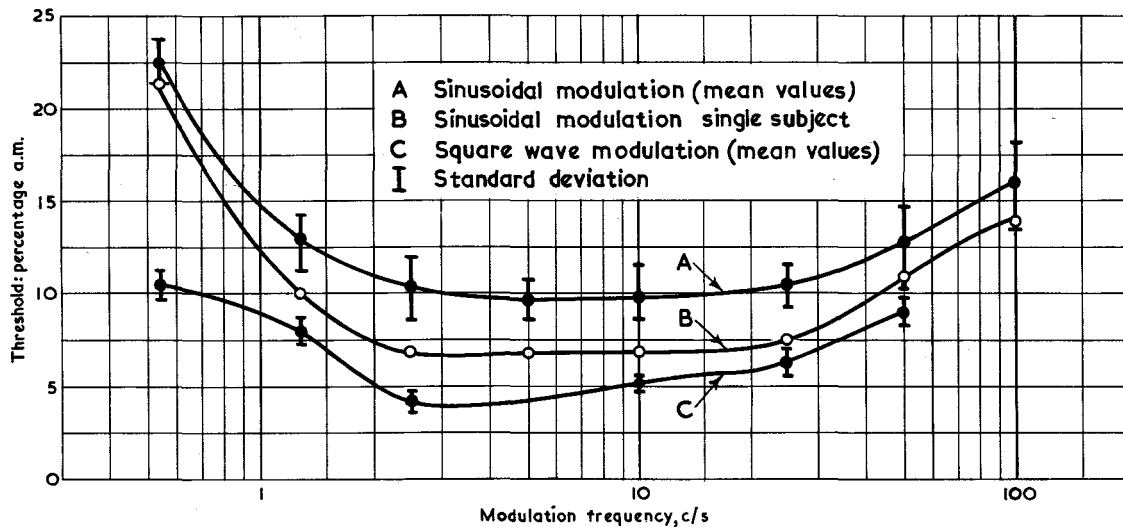


Fig. 12 - Variation of subjective threshold for amplitude modulation of piano programme heard through loudspeaker (LSU/10)

Curve B, taken under the same conditions, is for one exceptional subject with musical interests, who was found throughout various tests to possess a very acute discrimination. It is interesting to note that while these results fall within the deviations of the main group at the extreme frequencies, they are some 30% less in the 2 c/s to 20 c/s zone.

Curve C shows the mean results of the whole group when the programme was modulated by square-wave transitions. The shape is similar but the threshold is, as would be expected, depressed, particularly in the 0.5 c/s region. The value there of 10.5% corresponds to about 1 db change, popularly supposed to be the least noticeable level change. The reduction in threshold is accompanied by an increase in ease of judgement.

7.5. Piano Programme: H.F. Sinusoidal and Noise Modulation.

The great reduction of threshold found when tone is modulated at frequencies beyond a few hundred cycles, suggested that a similar state of affairs may be true for programme material. A test was, therefore, carried out with one of the more sensitive observers from the previous amplitude modulation experiments and the results, shown by Curve A, Fig. 13, confirmed the effect. In general the threshold is, as might be expected, considerably greater than for a pure tone, but the general shape of curve is similar, apart from the absence of zones of beating. In the 5 to 6 kc/s modulation region, however, the results are almost identical to the pure tone case. The aural impression is as if each percussive note excites a small, tinkling bell, and it may not be aesthetically displeasing. It is a good example of the hiatus between threshold value and annoyance value of these parasitic effects—in fact the latter may be difficult to achieve in an age accustomed to synthetic music.

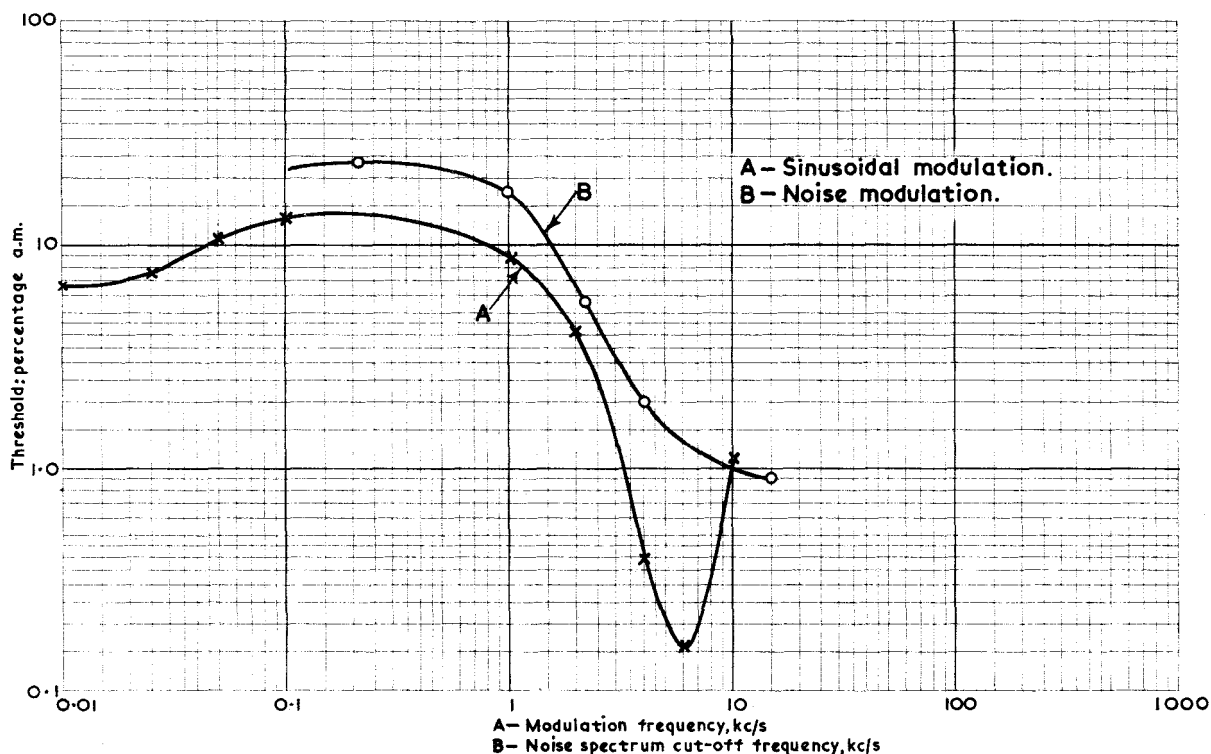


Fig. 13 - Variation of subjective threshold for amplitude modulation of piano programme

The state described could arise if, for instance, a magnetic tape was finely and regularly milled across its length—an unlikely state of affairs. Similar random variations of a medium undoubtedly do take place, however, and give rise to noise modulation. Hence the threshold level was examined when the programme was modulated with initially "white" noise restricted spectrally by a series of low-pass filters. The results shown by Curve B of Fig. 13 present threshold level (% amplitude modulation) versus the upper cut-off frequency of modulating noise. In no case do the noise components extend much below 20 c/s.

The results, except for the absence of a sharp dip at 6 kc/s, closely resemble those for H.F. sinusoidal modulation. When modulated by wide-band noise the programme is at once judged to have something of the character of poor tape recording, or of a slightly worn disc with a "gritty" accompaniment. The very marked decrease of the threshold when noise components extend beyond 1 kc/s is interesting, and indicates the value of polishing the surface of recording tapes, if by this process the occurrence of random high-frequency amplitude modulations are reduced. In a tape recorded at 15 inches/second, for example, undulations have a most serious effect when they occur at intervals shorter than 0.015 inches; undulations occurring at greater intervals would need some ten times the amplitude to have so severe an effect, unless they were of the order of a foot in length. The latter order would again bring the effect into the lower danger region of 2 to 3 c/s, where aural sensitivity to all types of fluctuations seems to be enhanced.

8. RESULTS OF FREQUENCY MODULATION TESTS.

8.1. Pure Tone: Sinusoidal Modulation.

The perception of frequency fluctuations is dependent on what is often called the differential pitch sensitivity of the ear. This sensitivity depends, however, on the conditions of experiment. The classic experiments of Shower and Biddulph⁷ were carried out using an arbitrary waveform in the sensitive zone of 2 or 3 c/s for modulating the pure test tones which were then presented through headphones at various intensity levels. The present experiments are intended to cover a wider range of modulating frequencies and waveforms at a fixed intensity level.

The mean results for a group of twelve subjects (engineers, age 30 to 40 years) using headphones are shown in Fig. 14. The results indicate the peak fractional frequency deviation just detectable at frequencies in the audio range, with the modulation frequency as parameter. The results, expressed in this manner, show that much smaller percentages of frequency shift than of amplitude changes (Fig. 9) are noticeable. Otherwise there are features in common, however, such as the increase of threshold at low frequencies and the sensitive region around 1 kc/s and for modulation frequencies of 2 or 3 c/s. There is also the zone of beating when 100 c/s is auto-modulated and a threshold maximum around 50 c/s modulation frequency. The standard error of means was of the order of 5% except at extreme values of both parameters, where it was double this value.

The results were extended at the single test frequency of 1 kc/s to cover modulation frequencies beyond 100 c/s. The results of Fig. 15 are for a single subject, the ordinate being modulation index and not fractional frequency deviation.

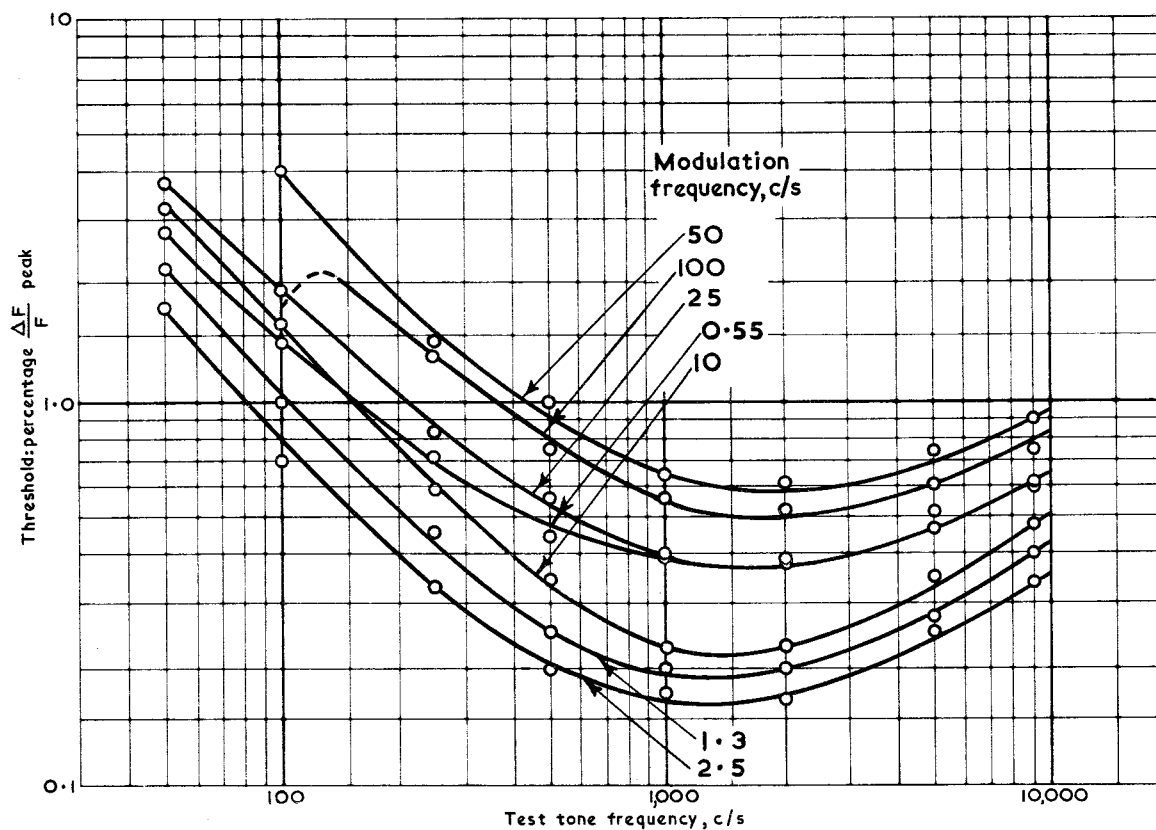


Fig. 14 - Variation of subjective threshold for sinusoidal frequency modulation of pure tone heard through headphones

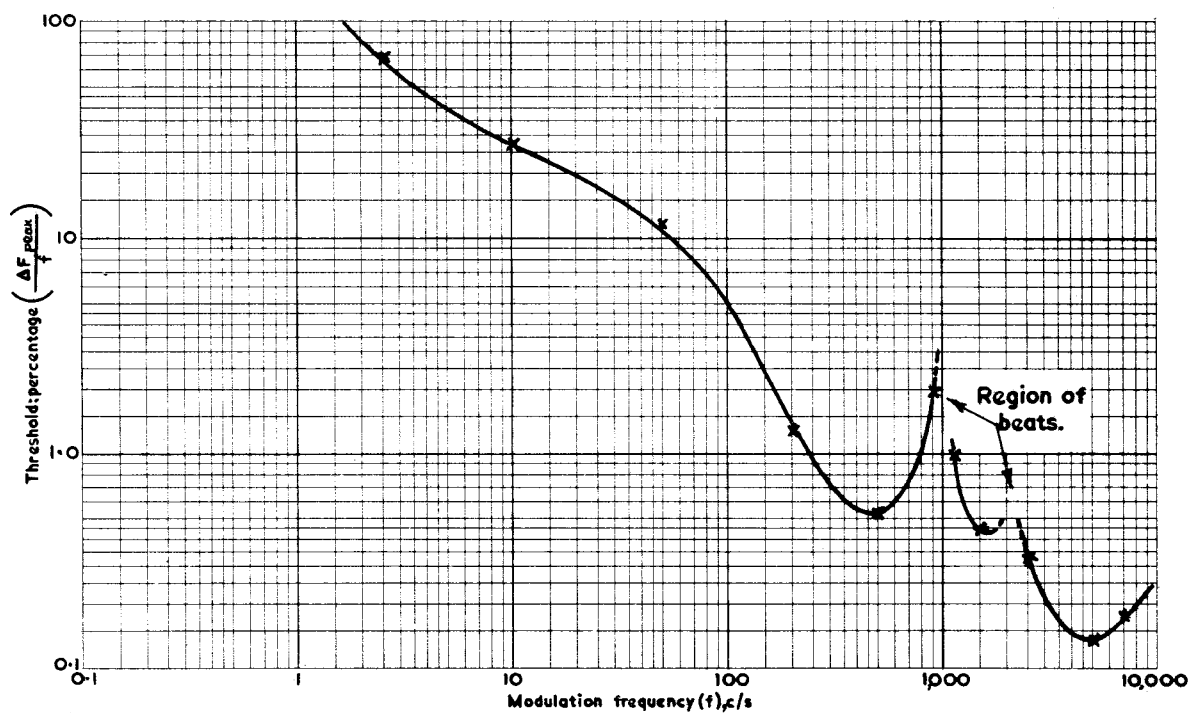


Fig. 15 - Variation of subjective threshold for sinusoidal frequency modulation of 1 kc/s tone heard through headphones

A comparison with Fig. 10 reveals the interesting fact that for modulation frequencies greater than 100 c/s, the threshold value of modulation indices is essentially the same for both amplitude and frequency modulation.

When a tone of varying pitch is heard through a loudspeaker in a normally reverberant room, instead of through headphones, the aural discrimination is affected by the tendency of the enclosure to form standing-wave patterns. In order to establish the rough order of magnitude of the smallest frequency excursion that could be detected under these conditions, an experiment was performed in the listening room at Nightingale Square using a frequency modulated tone fed to the loudspeaker. Every

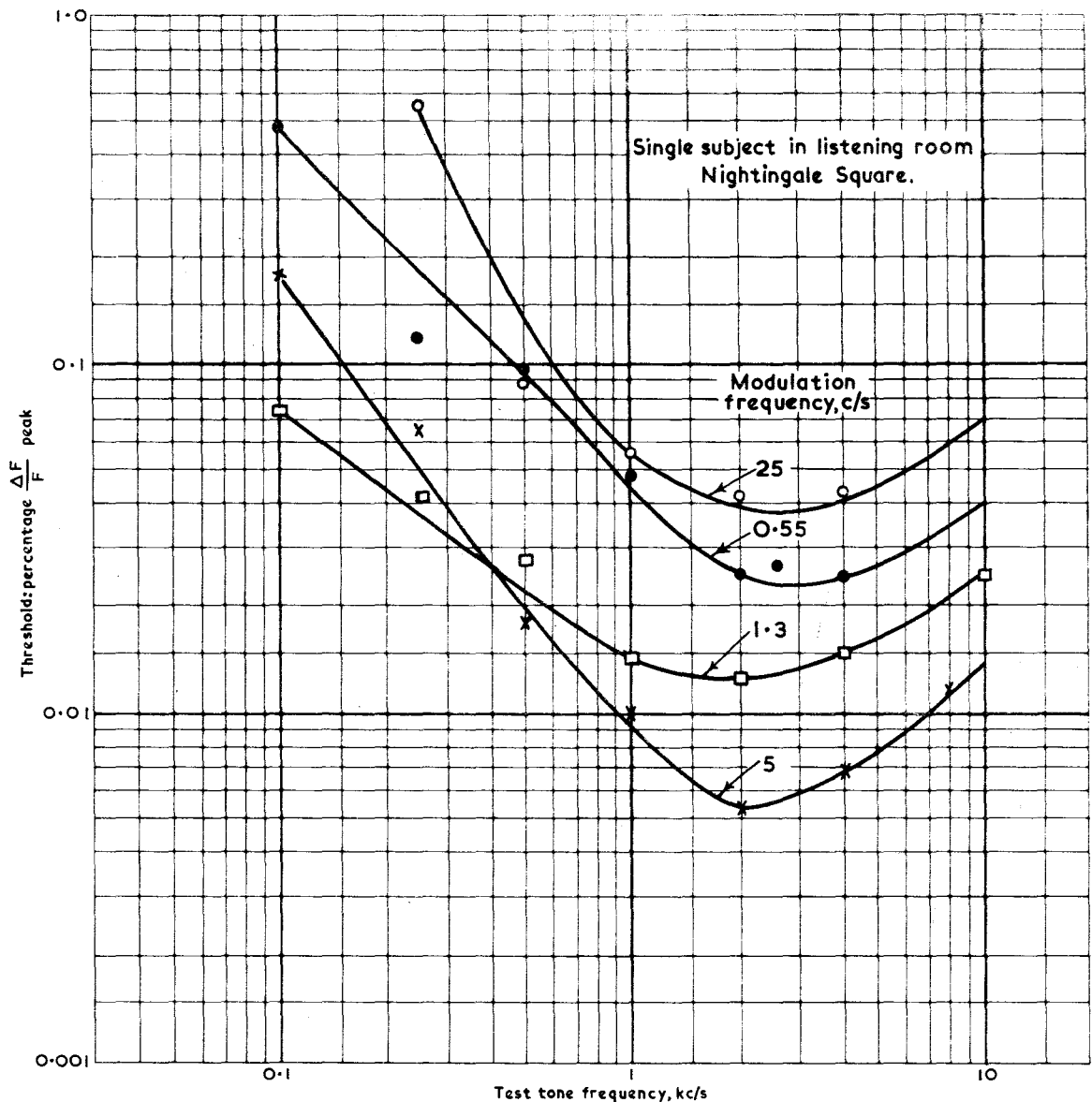


Fig. 16 - Variation of subjective threshold for sinusoidal frequency modulation of pure tone

conceivable device to enhance detection was employed in the experiments. For each particular test tone and modulation frequency the subject sought a position in the room where detection was enhanced either by the position of a nodal point, reflections, or harmonics of the tone, etc. No accurate level of sound intensity can, therefore, be quoted for the experiments at the instant of the measurement, for this level depended entirely on the spatial position of the subject as well as the loudspeaker response. The average level of 1 kc/s in the centre of the room was, however, adjusted to 75 phons.

Rather surprisingly these single spot results yielded not unreasonable curves, which are shown in Fig. 16. Tendencies similar to those shown by previous results are in evidence, both as regards the raised threshold at higher modulation rates and the sensitive region in the 2 kc/s zone. The action of the room enclosure has been, in general, to reduce considerably the absolute threshold values.

8.2. Pure Tone: Square-Wave and Impulsive Modulation.

If the pitch of a tone is changed abruptly, a greater number of sidebands are produced than is the case for sinusoidal modulation, with consequent lowering of the subjective threshold. The experiments of the last section were, accordingly, repeated using square-wave modulation and impulsive modulation produced by "differentiating" the square waves in various degrees to produce exponential decays of time constant 40, 5 and 1 milliseconds. The results of Fig. 17 show that the threshold is

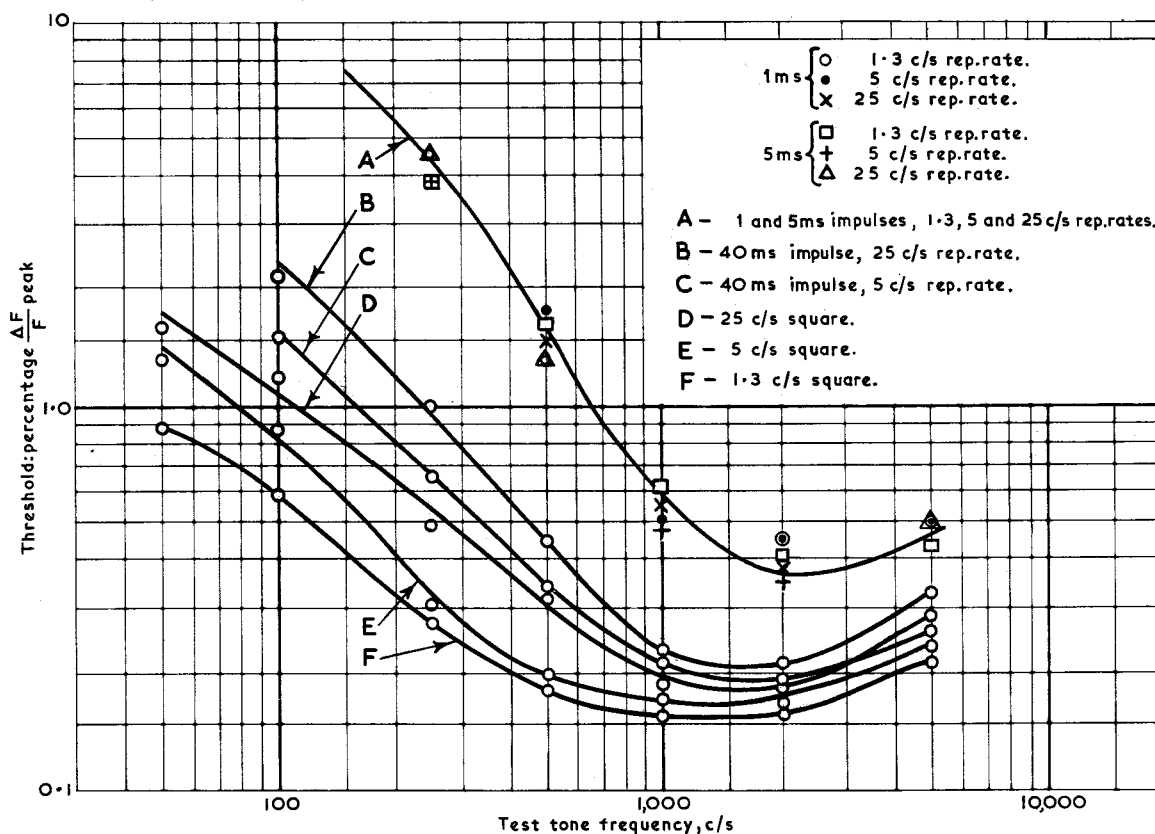


Fig. 17 - Variation of subjective threshold for square-wave and impulsive frequency modulation of pure tone heard through headphones

less dependent on the modulation frequency than is the case for sinusoidal modulation. When the transients decay with a 40 millisecond time-constant, the threshold rises and dependence on modulation frequency is further reduced. With a decay as fast as that corresponding to time constants of 1 to 5 milliseconds, the subjective threshold level is virtually independent of the time constant or repetition rate, between 1 c/s and 25 c/s at least. The variation with these parameters is of the order of experimental error, and a single representative curve has been drawn through the points. For square waves the standard error of the means was, in general, less than 5% in the 1 kc/s region, again rising to about twice the value in the 100 c/s region.

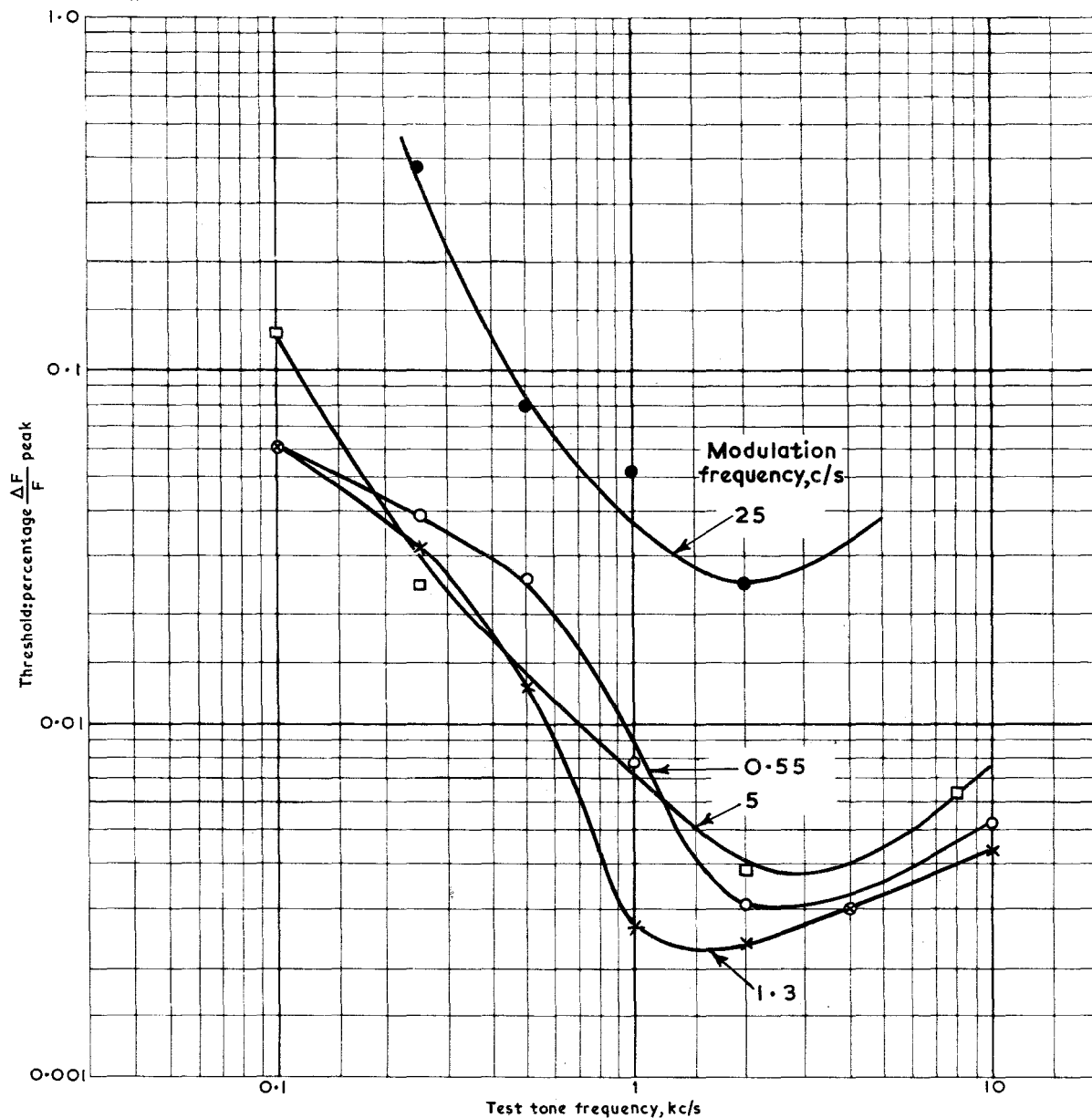


Fig. 18 - Variation of subjective threshold for square-wave frequency modulation of pure tone heard through loudspeaker LSU/10

That the perception of a series of "plops" does not markedly depend upon the rate of presentation seems reasonable from spectral considerations, for at 5 milliseconds the envelope of sideband energy is spread beyond 60 c/s, and the detail of line spectrum within it is unlikely to be of first-order consequence to the filter system of the ear.

The square-wave modulation was also repeated using a loudspeaker in the listening room exactly as described in the previous section, and the results of the single observations are shown in Fig. 18. It will be seen that detection of most minute changes is possible, the most extreme in this case being 0.024 c/s at 2 kc/s with a 1.3 c/s modulation frequency. It seems unlikely, therefore, that any recording of pure tone could be considered free of wow when heard in an enclosure. The change detected is not, of course, the frequency change per se, but the amplitude change caused by the modification of the standing-wave pattern of the enclosure.

8.3. Piano Programme: Sinusoidal Modulation.

Possibly the most important single class of fluctuation phenomena in recording systems are the periodic effects associated with non-centred discs or eccentric drive mechanisms. The word "wow" indeed suggests the almost sinusoidal change of speed at a frequency of 1.3 c/s which is commonly found in the reproduction of gramophone records. Present recording practice involves, however, a fundamental wow/flutter range from 0.5 c/s, found in "long-playing" and transcription work, up to 96 c/s which is produced in cine-film equipment. The experiments were, therefore, designed to cover this range as far as possible.

The subjects, about seventy in number, embraced a variety of ages, and included fourteen female subjects, sixteen subjects not associated with "sound engineering" in any way, and over a dozen engineers with specialist interest in the acoustic or audio-frequency field. The results of the experiments are shown in Fig. 19. The three upper curves are the mean values for the whole population (Curve B), for sixteen non-technical subjects (Curve A), and for fourteen female subjects (Curve C). The curves below, which are labelled correspondingly, show the standard deviations in each category. The distribution of threshold recognition versus fluctuation level was reasonably normal for the larger population. A histogram referring to 1.3 c/s modulation is shown in Fig. 20, superimposed on which is the corresponding normal distribution (not necessarily the best ordinate fit). The deviation of the two accompanying samples was calculated on a basis of normal distribution also, although these smaller samples were often skew towards the higher fluctuation values.

The curves exhibit features in common with those for pure tone, except that the most easily recognised modulation frequency has increased from about 3 c/s to between 5 c/s and 10 c/s, and the maximum which appeared around 50 c/s in pure tone tests is not now in evidence.

A new feature is the tendency to a slightly raised threshold at 2.5 c/s, which becomes progressively more marked with the general increase in threshold values in passing from Curves C to A. A similar tendency is manifest in the standard deviations, which peak at 2.5 c/s in all three cases. The reason for this effect is not understood, and it has been observed at varying times, with different piano

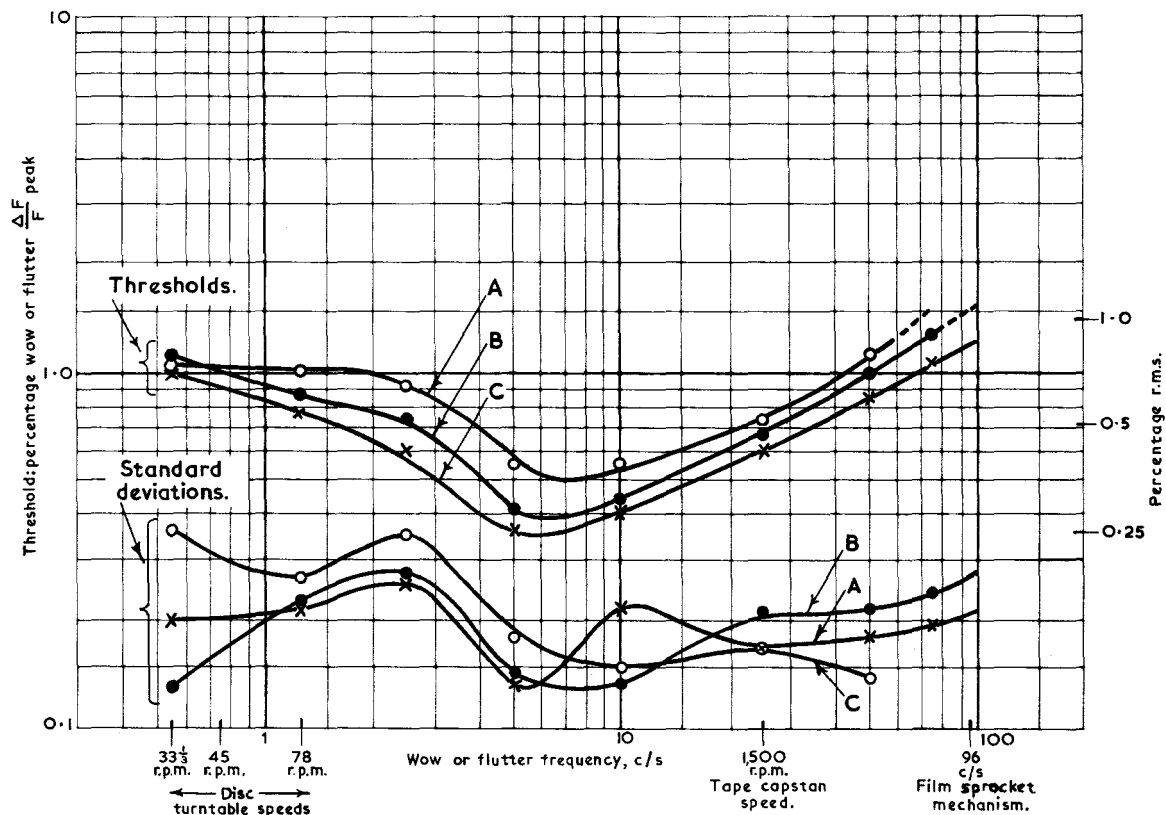


Fig. 19 - Variation of subjective threshold for sinusoidal frequency modulation of piano programme heard through loudspeaker LSU/10

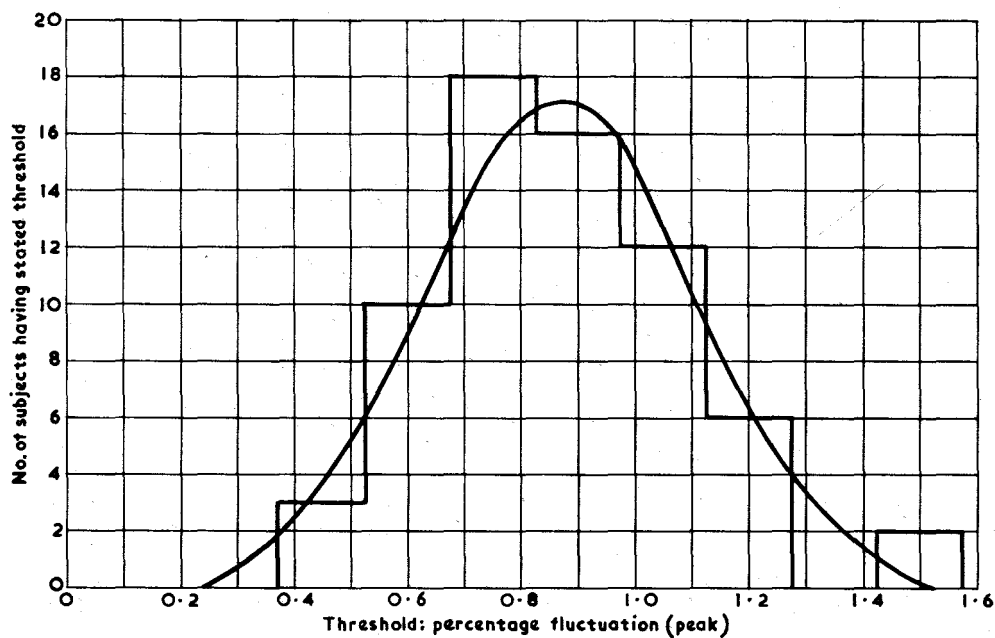


Fig. 20 - Distribution of subjective thresholds for 1.3 c/s sinusoidal frequency modulation of piano programme

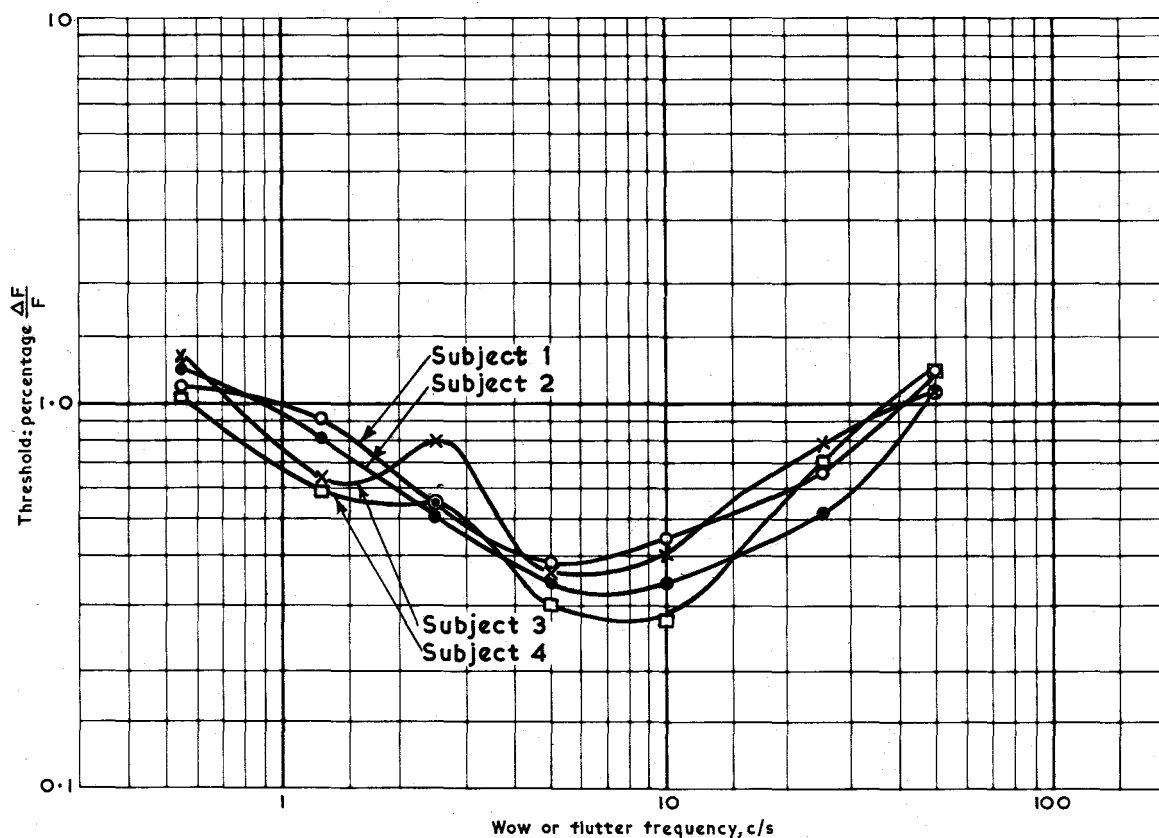


Fig. 21 - Variation of subjective threshold of four subjects for sinusoidal frequency modulation of piano programme (LSU/10)

programmes and in different listening rooms. Fig. 21, which shows the results of four individual observers, is interesting in this respect. It will be seen that two of the subjects do not exhibit the effect, whilst subject (3) shows it to a remarkable degree. There seems to be a minor tendency to increased sensitivity in the 5-10 c/s zone in subjects exhibiting this peculiarity.

This threshold increase at 2.5 c/s is possibly associated with rhythmical fluctuations of programme loudness at that frequency, which may confuse recognition to an extent dependent on musical training. As a matter of interest a rough check was made of the distribution of frequencies in the amplitude modulation envelope of "Am Meer" as edited and played under the standard conditions. The intensity level was recorded at very slow speed, and this signal, reproduced many times faster, was analysed by observing the output of filters tuned to the desired multiple of the rhythm sought. The results, Fig. 22, show the logarithm of average intensity changes plotted against particular rhythmic (fluctuation) frequencies in the programme level envelope, throughout the duration of the piece. There is a large fluctuation at 0.5 c/s, as was expected from the tempo and rhythm of the music, falling sharply as the frequency increased, but it will be noted that the level around 2.5 c/s is rather larger than would be expected from the general trend. The subjective thresholds for frequency fluctuations occurring with pure tone and amplitude modulated carriers would

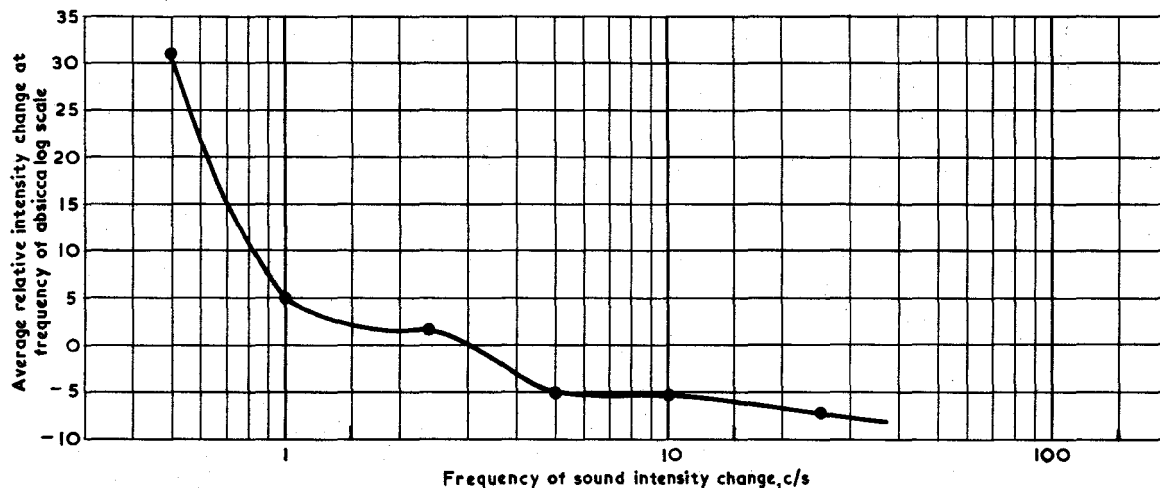


Fig. 22 - Average intensity changes associated with various rhythmic frequencies in "Am Meer"

be likely to exhibit differences correlating with this type of data, and it is tempting to assume such a correlation even if the modulated "carrier" were programme. This aspect has not, however, been pursued—all that can be said is that the result in the one case is not at variance with the idea.

The results in Fig. 19 exhibit a very similar trend in the means of samples A and C, despite the fact that with the relatively large deviations (of 30% order) there is little significance between means at a particular modulation frequency. The fact that the curves closely maintain a similar shape over the whole frequency range suggests greater significance. This was checked by combining the readings of the samples for all frequencies in the tests. To achieve this the means and deviations at a particular frequency were first expressed as percentages of the whole population mean. Three new sets including readings for seven frequencies up to 50 c/s were then derived, and their means and standard errors calculated for the combined number of measurements (i.e. for seven times the number of subjects in the original samples).

The result was:

Class	Mean %	Standard Deviation %	Standard Error %
A	116	39	3.9
B	100	32	1.5
C	87	33	3.4

The threshold difference between subjects of samples A and C, considering all modulation frequencies, clearly has a marked significance, although the possibility of unsuspected bias still exists with such relatively small samples.

8.4. Piano Programme: Square Wave Modulation.

The tests involving square-wave modulation were restricted to frequencies below 10 c/s, due to the limited response of the flutter generator. The group of

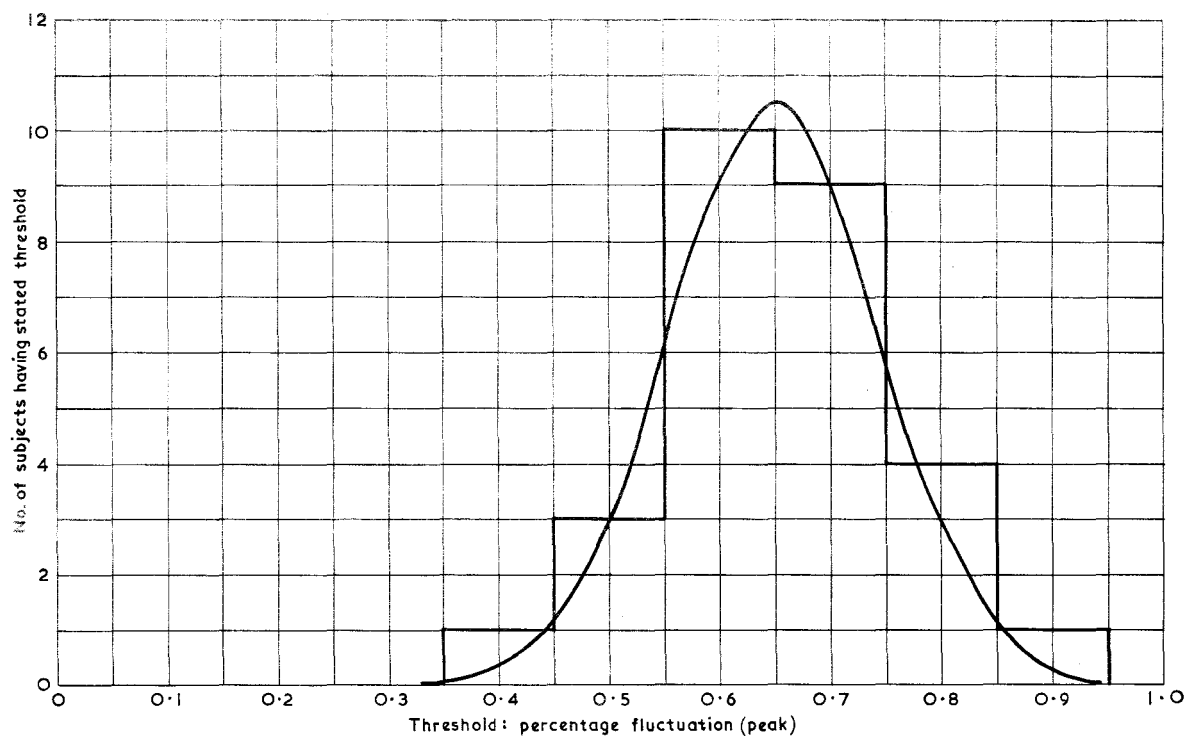


Fig. 23 - Distribution of subjective thresholds for 0.55 c/s square-wave frequency modulation of piano programme

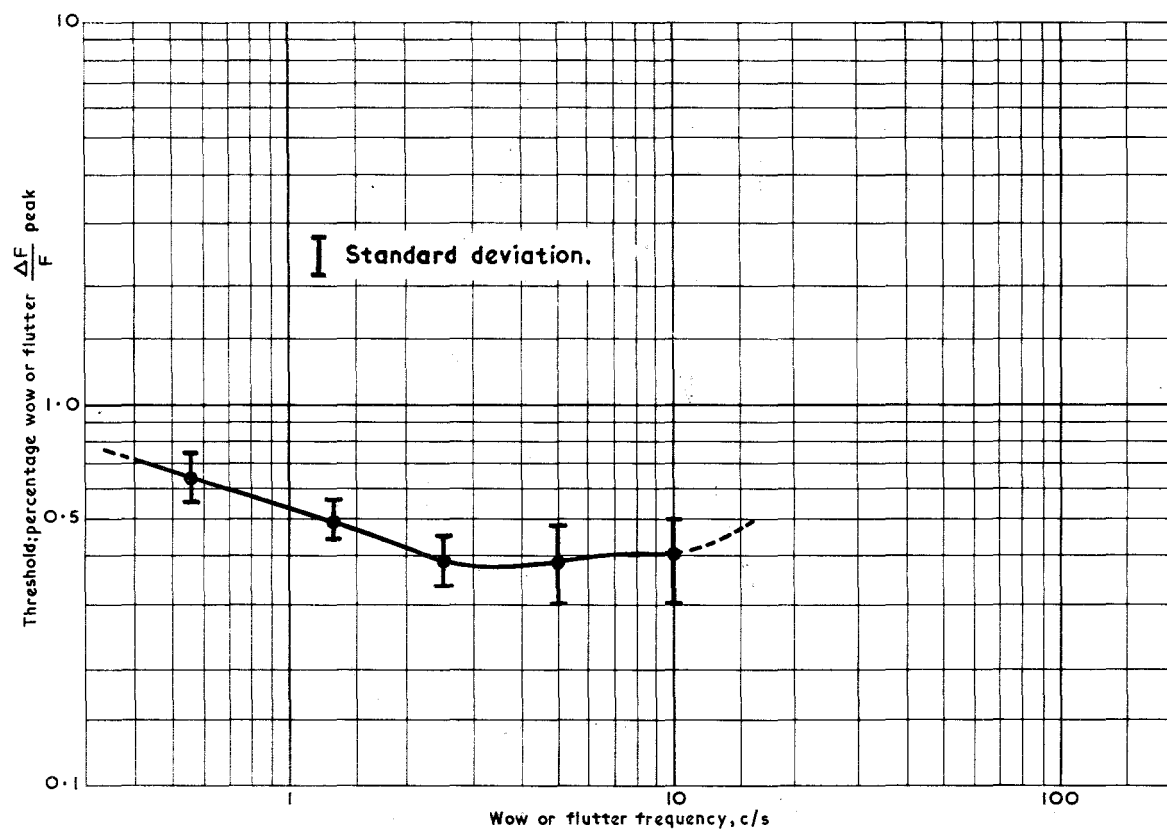


Fig. 24 - Variation of subjective threshold for square-wave frequency modulation of piano programme

subjects, 28 in number, was similar to population B of the previous section. Again the voting distributions were of fairly normal character and an example for 0.55 c/s modulation frequency is shown in Fig. 23.

The summarised results (Fig. 24) show that both the means and deviations are, in general, less than for the sinusoidal case, as expected. The depression of threshold is most marked at the lowest frequencies, which seems reasonable in that here the harmonics of modulation fall within a descending part of the sinusoidal component curve, tending to enhance recognition, whilst the reverse is true above 5 c/s where the harmonic modulation frequencies are less effective relative to the fundamental. With the spectrum restricted to a hundred cycles or so, the square and sine curves would be expected to become nearly identical around 10 or 20 c/s and this seems to be the case. If, of course, the "square" wave spectrum was unbounded, a further depression of threshold would be expected at all these frequencies, due to the presence of damaging harmonics at high audio frequencies (see Fig. 21) but such a state of affairs is virtually impossible in practical recording equipment.

8.5. Piano Programme: Impulsive Modulation.

The effect of a transient wow due to intermittent causes, such as tape adhesion, etc. can be roughly simulated by applying pulses to the flutter generator. The recognition of a single pulse is not a substantially different matter from the recognition of a train of pulses when the repetition rate is small. For convenience in the present experiments pulses were, therefore, presented every two seconds. The pulses had a decay time-constant of 80 milliseconds and their shape is shown in

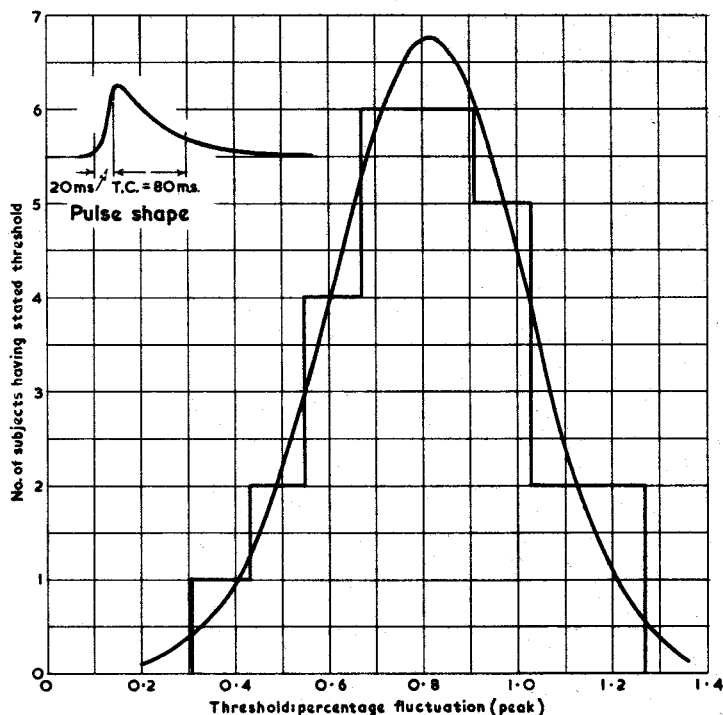


Fig. 25 - Distribution of subjective thresholds for impulsive frequency modulation of piano programme

Fig. 25, together with the results for the same group of 28 subjects. The voting histogram was found to approximate to a normal distribution with mean of 0.81% peak wow and a standard deviation of 0.21%.

8.6. Piano Programme: Random Modulation.

The residual wow of high-grade recording machines does not, in general, have obvious periodicity, but is of a stochastic nature. It is, therefore, of interest to examine the subjective effect of such fluctuations. The frequency fluctuations produced by the wow generator when fed by a source of random noise yielded a Poisson form of distribution, not unlike the form which occurs in practical machines. The relationship was described in more detail at the end of Section 5.1.

In the present experiment the piano programme was modulated in ten discrete steps up to 0.55% mean fluctuation, and the number of subjects whose threshold lay at a particular level is shown in the histogram of Fig. 26. The subjects tested comprised thirty-nine engineering and manual staff of various ages and both sexes. Four of the subjects did not detect the modulation at all, even at the maximum level presented. This, together with the shape of the voting histogram, suggests that the distribution is not a normal one. The distribution, in fact, appears to be of the same kind as that of the frequency fluctuations themselves, and Fig. 27 shows a plot of $\log_{10} [q(R)/R^2]$ against R , where R is the mean fluctuation level and $q(R)$ the number of subjects whose threshold falls at level R . The line $\log_{10} [q(R)/R^2] = 2.94 - 4.17R$, calculated by the method of least squares, is a reasonable fit to these points. The value of X (Section 5.1) for the distribution, derived from the slope of this line, is

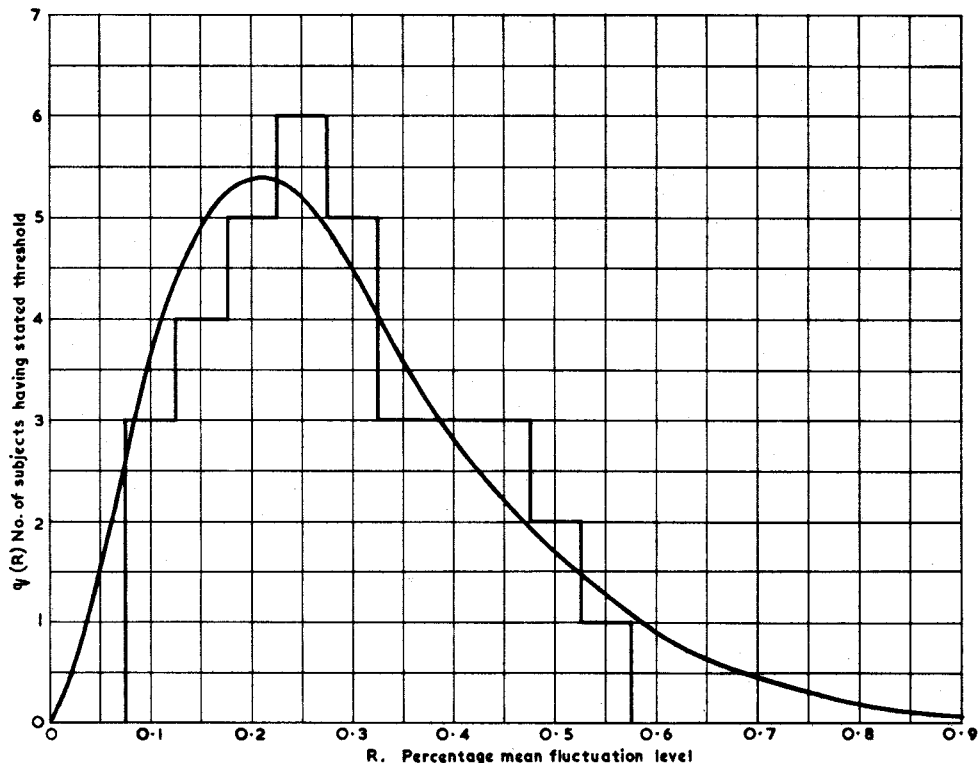


Fig. 26 - Distribution of subjective thresholds for random frequency modulation of piano programme

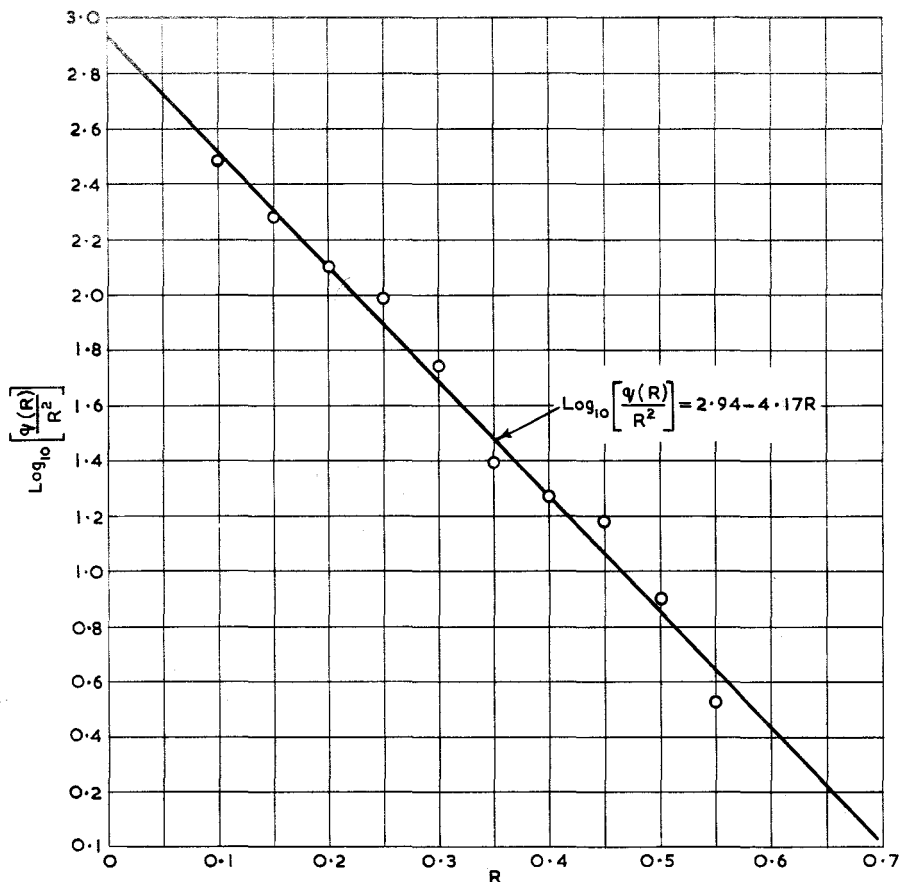


Fig. 27 - Distribution of experimental points about fitted Poisson relation

0.10. These constants admit the possibility of some three people from the group being unable to detect the fluctuations up to the level presented, which is in reasonable agreement with the facts. The mode (2X) is of the value 0.2% (mean fluctuation) and the mean of the distribution (3X) is 0.3% which, in a distribution of this type, implies peak fluctuations of the order of 0.8%—the value for pulse recognition found in the experiment described in the previous section.

8.7. Fluctuations in Other Programme Material.

From the fluctuation viewpoint the piano is possibly the most pure tonal generator available, in that its mechanical action prevents accompanying vibrato, and for this reason it was used almost exclusively in this investigation. Even the organ may exhibit amplitude vibrato controlled by the "tremulant" stop, and electronic keyboard instruments, such as the clavichord, have special vibrato generators. Any vibrato, either of intensity or pitch naturally tends to mask the fluctuations caused by recording systems. An example of the fluctuation threshold for light theatre-organ music is shown in Fig. 28 which summarises the results from nine engineers. The sound level, although occasionally peaking to 75 phons, in general tended to average 5 phons below this. In so far as comparison with Fig. 19, Curve B, is possible, the threshold is seen to be greater than that for piano programme, except at extreme flutter rates where it is of the same order.

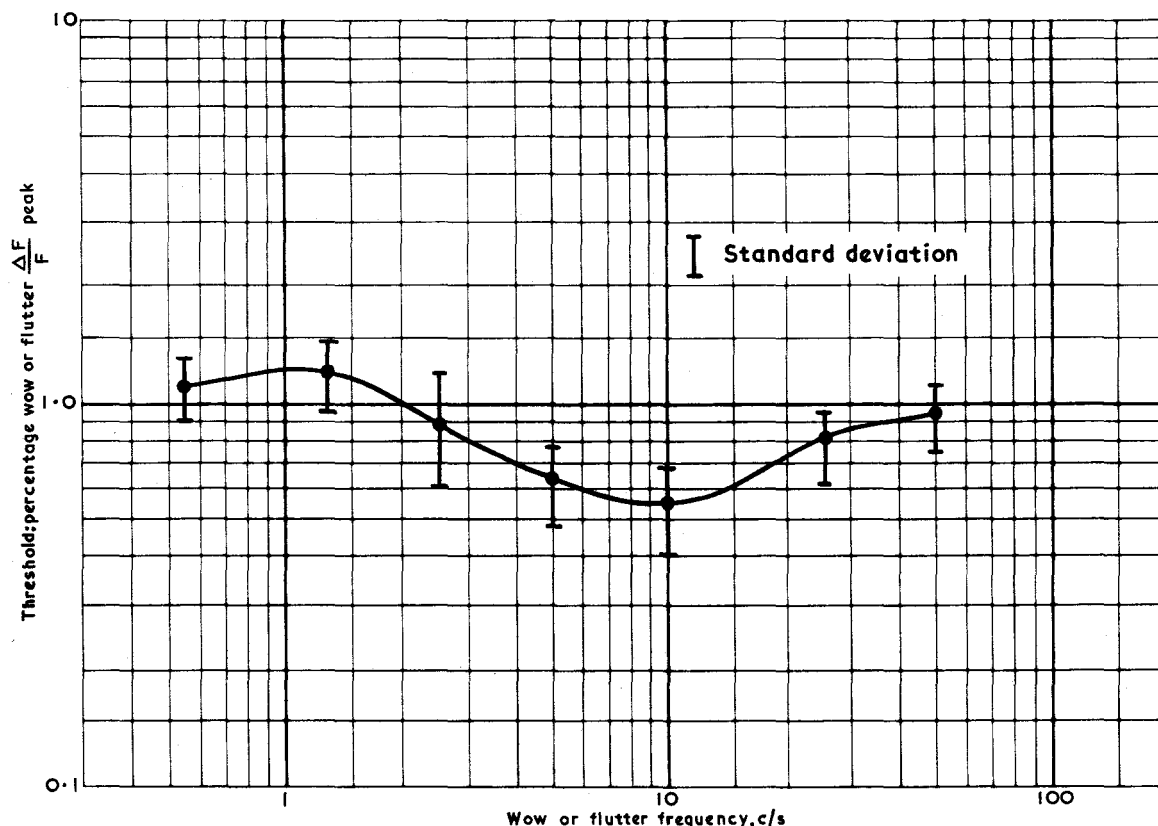


Fig. 28 - Variation of subjective threshold for sinusoidal frequency modulation of organ programme

The singing voice especially, has large amounts of vibrato—Caruso, for example, tended to have a 7 c/s pitch vibrato of about 3% peak. The transient nature of speech sounds, even unaccompanied by such vibrato, makes the recognition of frequency fluctuations difficult. The threshold value is about 2 or 3% peak at 1 c/s, and intelligibility is not affected by many times this amount. Seashore⁸, in discussing vibrato as a musical ornament, says, "Much of the most beautiful vibrato is below the threshold for vibrato hearing, and is perceived merely as tone quality. Individual differences in the capacity for hearing the vibrato are very large. In a normal population, one individual may be 50 or 100 times as keen as another in this hearing". The meaning of the first sentence is rather obscure, whilst the truth of the second seems unlikely in the light of the present experiments.

The perception of flutter in orchestral music is a very variable factor due to many changes of instruments and tone colours. Most wind instruments can, and do, produce vibrato, whilst violins as normally played are especially rich in the effect. Seashore quotes 6.5 c/s as being an average frequency for both the intensity and pitch vibrato of violinists with a magnitude of 4.4 db (66%) for the former and a quarter-tone for the latter. This is about half the value produced by singers. It is hardly surprising then that the flutter thresholds tend to be greater, on the average, for orchestral music than for piano music. No detailed quantitative data is available, and it would indeed be difficult to ascribe definite meaning to such information without embarking on a prolonged statistical treatment. In any event this would be unlikely to influence the design of driving systems for recording purposes.

9. DISCUSSION.

9.1. General.

The experiments described have provided information on the human discrimination of amplitude and frequency fluctuations, of various waveform and frequency, on tone, noise, and programme material. It is apparent that any "figure of merit" of a recording system from these viewpoints is markedly a function of the individual listener, of his environment, and of the recorded material. Although a considerable range of fluctuation magnitudes and frequencies have been explored in these experiments, it is pertinent to enquire to what extent these may be encountered in practical systems.

9.2. Amplitude Fluctuations in Practical Systems.

Dealing first with fluctuations of amplitude, all frequencies in the audio band would seem possible. Consider, for example, a recording delay system such as is used in the programme flutter generator and which is already in use for the production of artificial reverberation effects⁹. Any variation in the separation of the heads and the medium will cause amplitude modulation and, depending on the angular velocity of the drum, these variations may be at any frequency from below 1 c/s to about 25 c/s. Frequencies of amplitude modulation from here to about 1 kc/s may be produced by creep or distortion in the drum material or by "tool chatter" in the machining. Finally, small irregularities of surface finish or inhomogeneity of magnetic coating may produce fluctuations up to the highest audible frequencies. These high-frequency amplitude fluctuations need only be of 0.2% order to be audible; in fact some difficulty was at first experienced in eliminating such defects from the fluctuation generator.

The amplitude modulation resulting from out-of-contact working with drum systems may possess unfamiliar features. There may, for instance, be apparent frequency doubling of the eccentricity modulation depending on the bias setting in the recording head. Thus if the bias current is adjusted for maximum sensitivity at the mean separation, then both the increase and the decrease of spacing will reduce the recorded level. Also, the reproduced signal is proportional to $\exp(-2\pi d/\lambda)$, where d is separation between the head and the medium and λ is the recorded wavelength. Hence the amplitude modulation due to eccentricity is frequency selective, the lower frequency components of a programme being relatively unaffected. This feature requires consideration when assessing the subjective effect on programme on the basis of measurements at a single frequency, which is common practice.

9.3. Frequency Fluctuations in Practical Systems.

As regards frequency fluctuations, the fundamental range from about 0.5 c/s to 100 c/s is well known, and fluctuations of about 0.1 c/s or below, known as "drift", can be dismissed from this consideration as subjectively unimportant unless the range of pitch wanders as much as a semitone for some period, when the possessor of absolute-pitch would immediately complain. Fluctuations of frequency in excess of 200 c/s are not likely to be generated to any extent by existing recording systems because of the values of inertias and compliances generally involved in the mechanical systems. There are however some exceptions, for in magnetic recording systems the

mechanical constants of the tape are such that it may be excited by Coulomb friction, or random torques, into longitudinal resonance at frequencies of 1 kc/s or above, and analogous excitations of the pick-up in a disk system are theoretically possible. Such effects are unlikely to produce speed changes greater than 1%.

9.4. The Composite Curve of Frequency Fluctuation Thresholds.

The subjective threshold for fluctuations at frequencies greater than 100 c/s was not measured on programme material due to the limitations of the generator, but it is possible to assess its magnitude from other data. It has been observed, in the case of pure tone, that the threshold values of modulation indices for both amplitude and frequency fluctuations are virtually identical for modulation frequencies greater than 100 c/s or so (Fig. 10 and 15). From a spectral viewpoint this is hardly surprising, for in this range the identity of modulation indices expresses also the identity of energy distribution in the two principal sidebands, and the aural lack of phase-consciousness fails to resolve any difference. If we assume similar identity for the aggregate of frequencies in a programme source, the frequency-modulation threshold index will follow the amplitude-modulation index for programme (Fig. 13, Curve A) above 100 c/s, and so ΔF , the equivalent frequency excursion, may be derived from the index value by multiplying by $\phi/2\pi$, the modulation frequency, at any point. The new curve of relative ΔF excursion above 100 c/s can then be normalised to produce the value actually observed at 100 c/s, and the composite threshold curve for the entire frequency range is as shown in Fig. 29. A slight discontinuity of slope is noticeable at the 100 c/s junction, but no refined curve fitting was attempted as only an order of magnitude may be expected from this extrapolation.

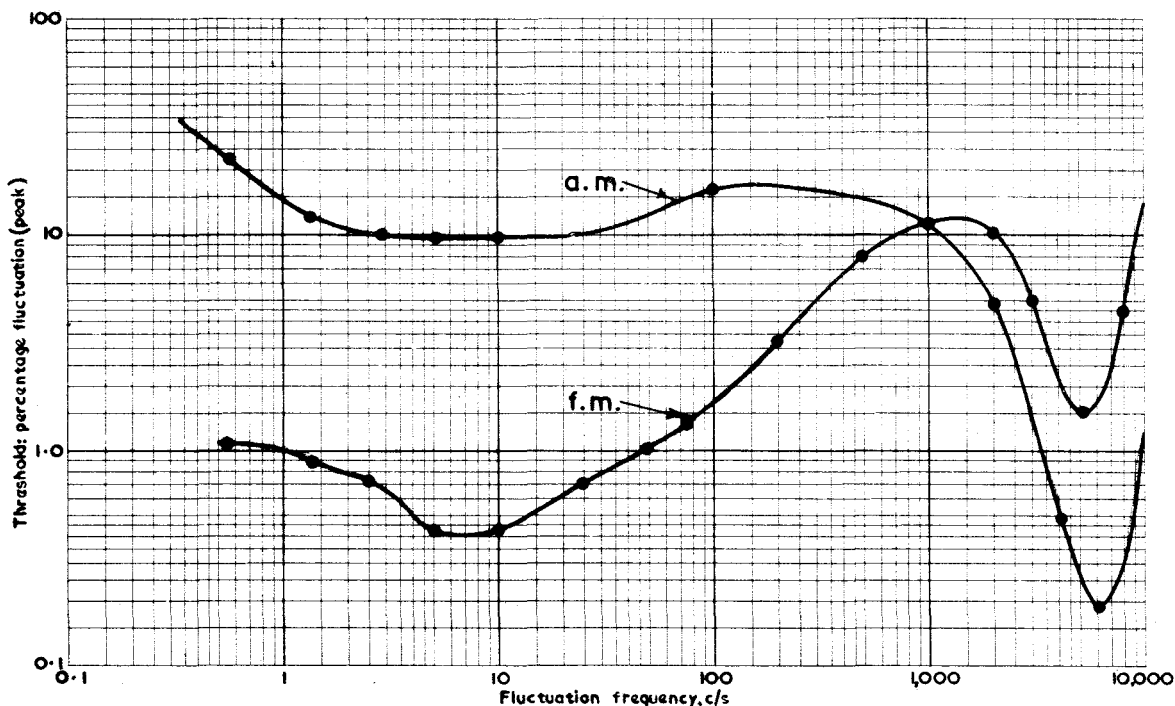


Fig. 29 - Composite curves of measured fluctuation thresholds (A.M. and F.M.)
on piano programme

The frequency modulation curve of Fig. 29 suggests that a minimum of 1.5% flutter is necessary to create a subjective effect above 100 c/s, and about 10% is necessary in the region of 1 kc/s. There would seem to be little object, then, in extending the range of flutter measurement beyond a few hundred cycles, in relation to programme recording systems as known at the present time.

9.5. The Specification of Tolerable Frequency Fluctuations.

Turning to the problem of specifying tolerable fluctuations from the subjective viewpoint, it is clear that some practical compromise is necessary, for if we insisted on listening to, say, 2 kc/s steady tone in a room, the permissible peak fluctuation could not exceed the order of 0.005%. A reasonable compromise would be to consider the system fluctuations in the light of the more stringent programme conditions which occur most frequently in practice—such as the piano programme used in the present experiments. A machine satisfactory on that basis would be adequate for organ and orchestral programmes, and virtually perfect for speech and any unusual transient effects. In other words, in the specification of tolerable fluctuations, and in their instrumental measurement, due weight should be given to experimental findings (such as the present ones) which reveal the (unwanted) modulation frequencies to which the ear is most sensitive and the type of programme on which these defects are most evident. A scheme for carrying out such subjectively-weighted measurements will be discussed, but it must be realised that it may require supplementing under special conditions. It is hoped, when such special cases arise, that suitable deductions may be made from the various curves given here which cover the subjective effects on tone and noise. Suppose, for instance, that a speech recording is taken on a recorder with 1.3 c/s wow via a radio link, which introduces a 9 kc/s whistle. The presence of such a whistle might reduce the threshold wow level by a factor of some 50 : 1, for Fig. 16 shows that about 0.02% wow could be detected on the whistle alone under these conditions. The 9 kc/s level would be below the 75 phon reference level used here, but as the amplitude-modulation threshold only rises by a factor of between 2 and 3 for a loudness level reduction from 80 to 30 phons, and the frequency-modulation threshold rises by a factor of about 2 for the same change, one must suppose the wow threshold to be little more than double the 0.02% value. Thus the tolerable level of 2% wow for speech alone is reduced, in this case, to approximately 0.04%. It may be argued that a modulation of existing distortion is of no concern, but the example serves to illustrate the points at issue. In a similar manner amplitude modulation of background noise may indicate the presence of fluctuations which would not be detected on programme alone. With such limitations in mind, we proceed to an examination of the tolerable fluctuations on programme, special regard being paid to the formulation of a practical measuring scheme.

9.6. A Universal Weighting Curve for Frequency Fluctuation Measurements.

The ideal subjectively-weighted measuring device should, when threshold value is obtained, indicate the same value irrespective of the type and frequency of the fluctuation to be measured. It will be shown that the results given here, as summarised by Fig. 29, indicate how a good approximation to this ideal device may be obtained. Consider, firstly, a single component of sinusoidal wow or flutter in the range 0.5 c/s to 200 c/s. If any system fluctuations are measured by a discriminator giving faithful conversion in this range, and the output envelope is equalised by a network whose frequency response is the inverse of the frequency modulation curve of

Fig. 29, then this output will be constant at threshold value throughout the frequency range. If there is more than one fluctuation component, the system will still suffice and the threshold is measured by the peak value of the weighted sum. If, for example, two equal components are close in frequency, giving a slowly beating fluctuation, the threshold is essentially at the peak value of the mixture that equals the threshold of one tone alone. If the frequencies are well separated the one of lower threshold is, of course, the predominant factor. Thus for a 1 kc/s tone, threshold is 0.2% at 1.3 c/s frequency of flutter and 0.4% at 25 c/s frequency of flutter, whilst experiment shows that the threshold for an equal mixture is 0.26%. On the equalising system suggested, half weight is given to the 25 c/s component and we have for the relative outputs:

$$\begin{aligned} 1.3 \text{ c/s} &:- 0.2 \times 1 = 0.2 \\ 25 \text{ c/s} &:- 0.4 \times \frac{1}{2} = 0.2 \\ \text{Mixture } \left\{ \begin{array}{l} 1.3 \text{ c/s} :- 0.13 \times 1 \\ +25 \text{ c/s} :- 0.13 \times \frac{1}{2} \end{array} \right\} &= 0.195 \end{aligned}$$

which, for practical purposes, is an identical reading in each case.

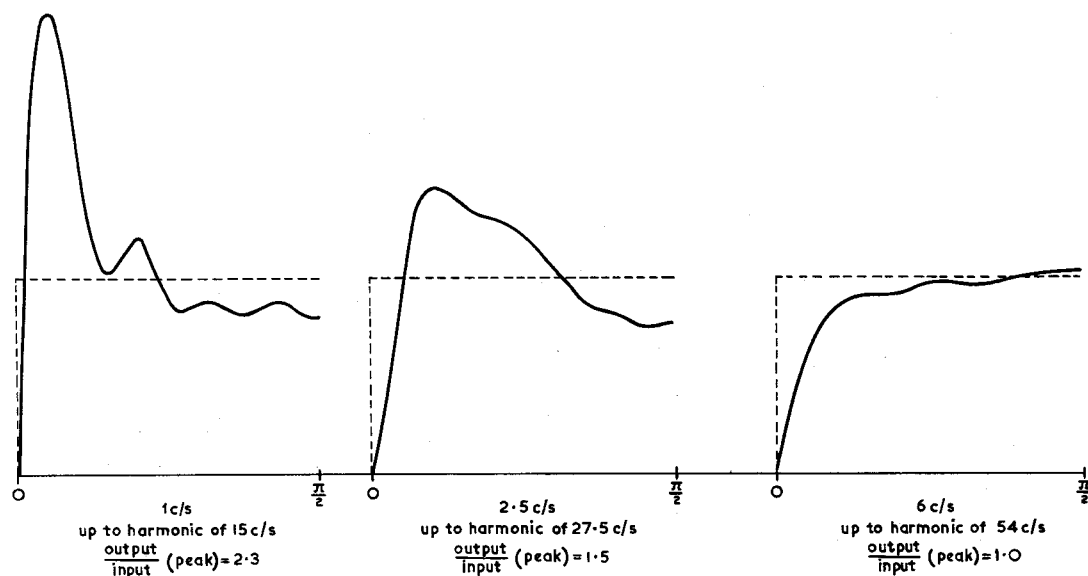


Fig. 30 - Response of suggested weighting network to squarewave input

Consider now the effect of other periodic waveforms. Experimental data has been obtained on square-wave modulation up to a frequency of 10 c/s. If square waves are applied to the weighting network (the inverse of the frequency modulation curve in Fig. 29) the relative peak value emerging will not be substantially altered when the rate is in the region of 6 c/s or above, but when the frequency is below this an overshoot takes place due to the selective increase of harmonic content. The amount of overshoot relative to the input is shown in Fig. 30, for the cases of 1 c/s, 2.5 c/s and 6 c/s. The graphs are approximate and show the effect of weighting the stated number of harmonics without relative phase differences. Constant peak-output of the network is thus achieved when the input signal is reduced by the ratio of the peak output to input, that is by approximately 1, 1.5, and 2.3. The subjective measuring device then yields the square-wave modulation threshold as being reduced by these

amounts on that of the sinusoidal case and the result is, to the first significant figure:

	Threshold	1 c/s	2.5 c/s	6 c/s
Sine	(Experimental)	0.9	0.7	0.4
Square	(Experimental)	0.5	0.4	0.4
Square	(Derived by weighting)	0.4	0.5	0.4

The agreement between experiment and the proposed measuring system is thus moderate—there is over-compensation below about 1.5 c/s, and under-compensation above. It is possible that a suitable phase characteristic would correct this to some extent. However, the probability of a practical measuring device being confronted with so stringent a test is not very great.

Finally, consider the effect of the proposed weighting on the continuous spectra which are involved in the measurement of random and pulse modulations. The experiments yielded a mean threshold-value of 0.8% peak in either case, the bandwidth of the modulation being restricted to 100 c/s by the design of the flutter generator. It was shown in Section 5.1 that the statistical distribution of amplitudes in the random frequency fluctuation tests obeyed a Poisson relation. It may be shown from this that amplitudes up to 0.8% peak comprise 99% of the fluctuations occurring in a random set of this nature. Assuming the maximum weighting to be unity at 6 c/s, where the sinusoidal threshold is 0.4% peak, the ratio of peak output to input should be 0.5 in the two cases if true threshold is to be indicated. An approximate analysis suggests that such is the case. The modulating pulse, of form e^{-kt} , ($t > 0$), has a Fourier transform $(k + j\omega)^{-1}$ and the spectrum amplitude $|k + j\omega|^{-1}$ is shown as Curve C in Fig. 31 for an 80-millisecond time-constant. The spectrum of the output signal (Curve D) has been weighted by the inverse-threshold Curve B. The relative frequency spread of Curves C and D are similar, and by the Fourier Integral Energy Theorem we may expect that the ratio of peak output-to-input will be roughly proportional to the ratio of areas beneath Curves D and C, which is about 0.6. Assuming a spectrum which is level up to 100 c/s for the random fluctuations, weighting Curve B is the relative amplitude spectrum of noise output, and the peak output/input level will be the ratio of areas beneath Curve B and the line A, up to 100 c/s, which is about 0.4.

The suggested frequency weighting thus yields a constant indication of subjective threshold value to within about 20% in all cases considered, which in view of the variations of judgment involved in such phenomena may be considered a reasonable accuracy. An absolute calibration of the system is required and this could be performed at say, 6 c/s, taking as standard the mean value of 0.4%. For critical standards it might be preferable to assume a value of the mean less twice the value of the standard deviation, so that fewer than 5% of listeners would be expected to notice a defect in a recording up to standard. This value corresponds to 0.14% peak, or 0.1% R.M.S. for a sine wave, which is a value that for many years has been accepted as a good criterion for total R.M.S. flutter in recording systems.

9.7. A Universal Weighting Curve for Amplitude Fluctuation Measurements.

A similar method of weighting could be employed for amplitude fluctuations, using a curve which is the inverse of the amplitude-modulation threshold curve for

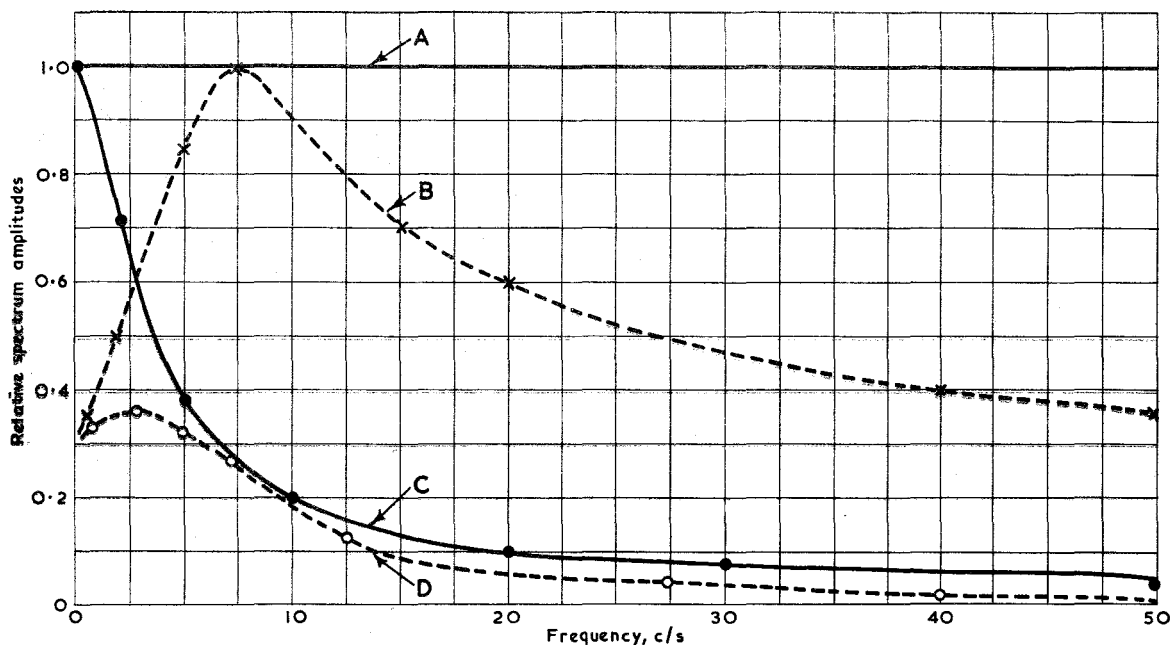


Fig. 31 - Determination of weighted spectra for random and impulsive fluctuations

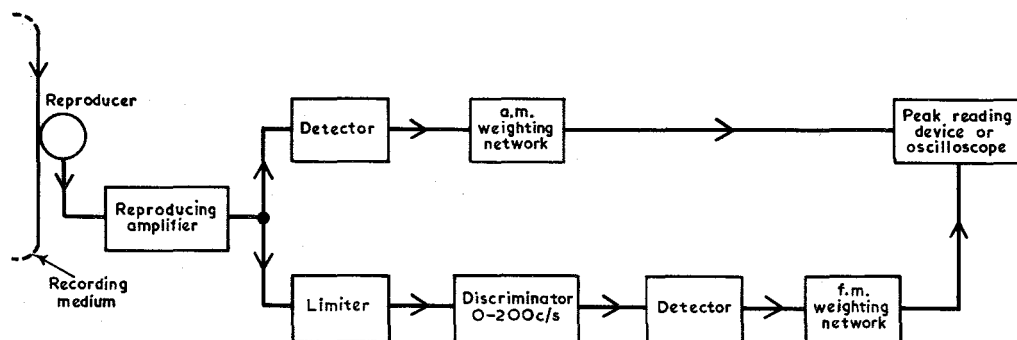


Fig. 32 - Schematic diagram of measuring system for frequency and amplitude flutter

piano programme, which is also reproduced for the whole frequency range in Fig. 29. If information about noise modulation is not required, the peak around 6 kc/s could be removed. If it is retained, care must be taken to exclude other sources of noise, such as thermal and shot noise, which are still present when the recording medium is at rest.

9.8. Measuring System for Frequency and Amplitude Fluctuations.

A block diagram of the proposed measuring system for both frequency and amplitude flutter is shown in Fig. 32. In practice a tone from the reproducing chain,

in which frequency or amplitude modulation exists, would be fed, in turn, into the frequency and amplitude modulation chains and a reading obtained on the peak reading device, or oscilloscope, which had been previously calibrated in absolute terms. A tone of frequency 3kc/s, which is becoming standard for such tests, could be employed.

10. EXISTING STANDARDS.

The information on "Measurement of Frequency Variation" in British Standard Memorandum B.S. 1988:1953 is not generally at variance with the information gained here but requires supplementing. It is there recognised that an unweighted R.M.S. summation is of limited use. The British Standard 1928:1953 on "Lateral Cut Gramophone Records and Direct Recordings", specifies tolerances which in all cases permit only fluctuations below the subjective thresholds determined in this investigation. The American I.R.E. Standards on Methods of Determining Flutter Content, 1954¹⁰, favour R.M.S. measurement, defining "percentage flutter" as the "R.M.S. deviation from average frequency expressed as a percentage of the average frequency". Mention is made of a "Flutter Index" which is "a measure of perceptibility of Frequency Modulation of a single tone". This index, for continuous tones in a moderately live room, is given as

$$I = x \cdot \Delta f / r$$

where Δf is the R.M.S. deviation of frequency, r is the flutter rate, and $x = r^2/30$ below 1 c/s, $x = r/30$ from 1 c/s to 5 c/s, $x = 1/6$ beyond 5 c/s. This would seem to be very approximate, and of limited significance on transient material such as programme.

11. CONCLUSION.

It appears, from the results and calculations set out in this report, that it is possible to define useful frequency-weighting curves for incorporation into chains measuring both amplitude and frequency modulation in practical recording systems. Providing that these chains have no inherent errors of their own, the use of these weighting curves should enable indications of equal subjective-threshold value to be obtained irrespective of the waveform or frequency of the unwanted modulation. The provision, noted in the last sentence, that the basic measuring chains should be free from inherent errors of their own is important and, in this respect, special caution is necessary in the design of discriminator circuits for use in the frequency-modulation chain. In the course of this investigation the properties of discriminators for use in the flutter-measuring chain have been investigated in some detail, and the results, together with recommendations which arise out of the work, will be included in a separate report dealing with measuring technique. It has been the purpose of the present report to establish tolerances and weighting curves based on the subjective aspects of undesired frequency and amplitude modulation. Having defined these factors it will be desirable also to make recommendations on the methods of measurement by which any future standards, based on the subjective findings, are brought into practice. The separate report will, therefore, form a corollary to the present one.

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13. APPENDIX—Mathematical Analysis of Flutter Generation.

Consider the recording system shown schematically in Fig. 33. The medium moves past a stationary recording head with a velocity compounded of constant velocity v_0 and a small unwanted variation $v \sin \omega t$, a representative component of inevitable traction irregularities. Suppose that there is a correlation between the recording and reproducing process in that the reproducing head is centred at a distance s from the recording head, with reproduction taking place at a time or order $T_0 = s/v_0$ later. Let there be some relative motion of the reproducing head (as in the flutter generating system described) which can be represented by a small velocity $v_1 \cos \omega_1 t$, thereby causing the separation of heads to be $s(t) = s + (v_1/\omega_1) \sin \omega_1 t$. It is proposed to determine the instantaneous frequency f_r which will be reproduced from the system under various conditions when a frequency f is fed into it.

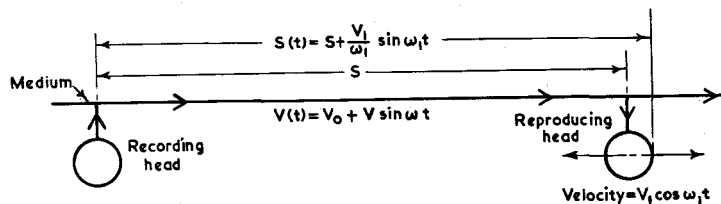


Fig. 33 - Mathematical relations in flutter generator

It is first necessary to determine T , the time at which reproduction takes place. Since both the velocity and distance traversed by a recorded element before reproduction are time dependent, T is given by the equation

$$s(t + T) = \int_t^{t+T} (v_0 + v \sin \omega t) dt$$

$$\therefore s + (v_1/\omega_1) \sin \omega_1(t + T) = v_0 T - (v/\omega) [\cos \omega(t + T) - \cos \omega t]$$

or

$$T - T_0 = T_1 \sin \omega_1(t + T) + T_2 [\cos \omega(t + T) - \cos \omega t] \quad (1)$$

where

$$T_0 = s/v_0 \quad T_1 = v_1/v_0 \omega_1 \quad T_2 = v/v_0 \omega$$

clearly

$$|T - T_0| \leq T_1 + 2T_2$$

Unless ω and ω_1 are very small, T_1 and T_2 are small quantities, since v/v_0 is of 1% order or less in any reasonable practical system and v_1/v_0 is of the same order in the flutter generator described in Section 3. Hence we shall neglect squares of these quantities and substitute T_0 for T in the right-hand side of Equation 1, yielding the approximation

$$T = T_0 + T_1 \sin \omega_1 (t + T_0) + T_2 [\cos \omega(t + T_0) - \cos \omega t] \quad (2)$$

A recorded signal $e_1 = \Phi(t)$ is reproduced as $e_0 = k\Phi(t - T)$, where k is some transfer constant, so that

$$e_0 = k\Phi\{t - T_0 - T_1 \sin \omega_1 (t + T_0) - T_2 [\cos \omega(t + T_0) - \cos \omega t]\}$$

The ratio of the instantaneous frequency f_r of output to instantaneous frequency f of input will be $d(t - T)/dt$ by the usual definition, so that in this case

$$f_r/f = 1 - \omega_1 T_1 \cos \omega_1 (t + T_0) + 2\omega T_2 \sin (\omega T_0/2) \cdot \cos \omega(t + T_0/2) \quad (3)$$

If s is made sufficiently small, $\sin (\omega T_0/2) \approx \omega T_0/2$

and

$$f_r/f = 1 + (\omega v s / v_0^2) \cos \omega(t + T_0/2) - (v_1/v_0) \cos \omega_1(t + T_0) \quad (4)$$

This is the equation which applies essentially to the high speed drum and moving reproducing head system used in the flutter generating system described.

In normal recording systems, however, the reproducing head is stationary, but the velocity of the medium varies. Here then $v_1 = 0$ and Equation 3 becomes

$$f_r/f = 1 + (2v/v_0) \sin(\omega s/2v_0) \cos \omega(t + T_0/2) \quad (5)$$

Thus there is no frequency change when $\omega s/v_0 = 0, 2\pi, 4\pi$, etc. and there is a maximum change when $\omega s/v_0 = \pi, 3\pi, 5\pi$, etc. The head spacing may then be arranged, in the limit, either to cancel or double a particular flutter frequency, although if the wow is complex it is clearly best to make the spacing as near zero as possible to achieve best overall cancellation. In the usual description of a frequency modulated wave an equation such as (5) can be regarded as expressing the "instantaneous frequency". The form in the case of frequency flutter is

$$f_r = f + \gamma f \cos \omega t \quad (6)$$

where the constant γ is independent of f .

The general expression for frequency modulation is of the form

$$e = e_0 \sin 2\pi \cdot (ft + [\Delta f/p] \sin pt)$$

where f is the carrier frequency, $p/2\pi$ is the modulation frequency, Δf is the frequency deviation. This yields for the instantaneous frequency

$$f_1 = f + \Delta f \cos pt \quad (7)$$

Comparing (6) and (7) it can be seen that for flutter

$$\Delta f = \gamma f$$

i.e. the deviation is proportional to the recorded frequency. It is this identity which prevents the simulation of wow or flutter in complex programme signals by means of heterodyne, or modulation systems. All such systems yield a deviation independent of the frequency of the signal components. The desired result can be achieved, however, by a memory device, which in the present work takes the form of a record on a magnetic drum.